

Exercice 3.20.

On donne : $P(-3; -2)$; $(\gamma) : x^2 + y^2 - 2x - 2y - 23 = 0$; $(d) : \begin{cases} x = k \\ y = 4 + 2k \end{cases}$, $k \in \mathbb{R}$

a) $\Rightarrow (\gamma) : (x - 1)^2 + (y - 1)^2 = 25 \Rightarrow C(1; 1)$ et $r = 5$ [u]

b) en éliminant les $k \Rightarrow (d) : 2x - y + 4 = 0$

c) • d coupe γ en deux points $\iff \delta(C; d) < r$ (voir page 104) :

$$\delta(C; d) = \frac{|2 \cdot 1 - 1 + 4|}{\sqrt{2^2 + (-1)^2}} = \frac{|5|}{\sqrt{5}} < 5$$

• $d \cap \gamma = \{P; Q\}$:

$$\begin{cases} (x - 1)^2 + (y - 1)^2 = 25 \\ 2x - y + 4 = 0 \end{cases} \iff \begin{cases} (x - 1)^2 + (y - 1)^2 = 25 \\ y = 2x + 4 \end{cases}$$

$$\Rightarrow (x - 1)^2 + (2x + 3)^2 = 25 \iff x^2 - 2x + 1 + 4x^2 + 12x + 9 - 25 = 0 \iff$$

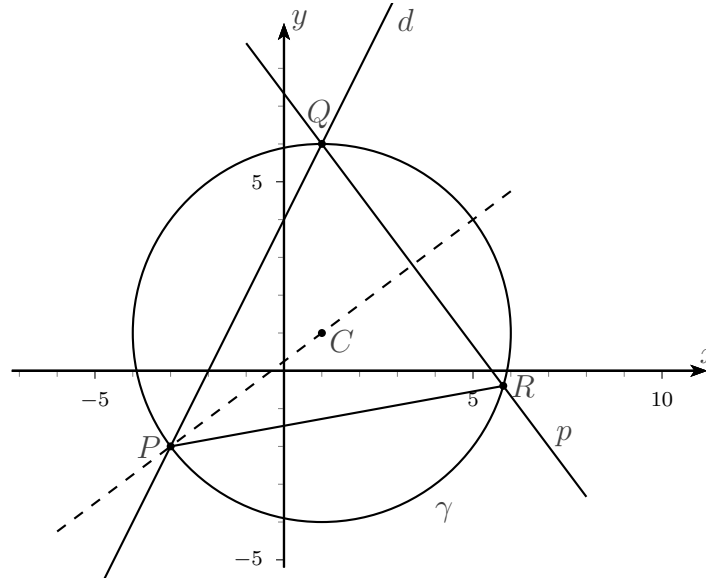
$$\iff 5x^2 + 10x - 15 = 0 \iff 5(x^2 + 2x - 3) = 0 \iff 5(x + 3)(x - 1) = 0 \Rightarrow x_1 = -3 \text{ ou } x_2 = 1$$

et $y_1 = -2$ ou $y_2 = 6$

$$\Rightarrow \boxed{P(-3; -2) ; Q(1; 6)}$$

d) Marche à suivre :

- 1) droite PC
- 2) $p \perp PC$ par Q
- 3) $p \cap \gamma = \{Q; R\}$
- 4) tracer PR



$$1) \overrightarrow{PC} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

$$2) p \perp PC \text{ par } Q \xrightarrow{\text{p.84}} (p) : 4x + 3y + c = 0 \text{ passe par } Q(1; 6) : \\ \Rightarrow 4 \cdot 1 + 3 \cdot 6 + c = 0 \iff 22 + c = 0 \iff c = -22 \\ \Rightarrow (p) : 4x + 3y - 22 = 0$$

$$3) p \cap \gamma = \{Q; R\} :$$

$$\begin{cases} (x-1)^2 + (y-1)^2 = 25 \\ 4x + 3y - 22 = 0 \end{cases} \iff \begin{cases} (x-1)^2 + (y-1)^2 = 25 \\ y = \frac{-4x+22}{3} \end{cases} \\ \Rightarrow (x-1)^2 + \left(\frac{-4x+19}{3}\right)^2 = 25 \iff x^2 - 2x + 1 + \frac{16x^2 - 152x + 361}{9} - 25 = 0 \iff \\ \iff 9x^2 - 18x + 9 + 16x^2 - 152x + 361 - 225 = 0 \iff 25x^2 - 170x + 145 = 0 \iff \\ \iff 5(5x^2 - 34x + 29) = 0 \iff 5(5x - 29)(x - 1) = 0 \Rightarrow x_1 = \frac{29}{5} \text{ ou } x_2 = 1 \\ \text{et } y_1 = -\frac{2}{5} \text{ ou } y_2 = 6$$

$$\Rightarrow \boxed{R\left(\frac{29}{5}; -\frac{2}{5}\right); Q(1; 6)}$$