

Exercice 3.11.

$$\text{a) } \lim_{x \rightarrow -1} \frac{2x^3 - 2x}{3x^3 - 6x^2 - 9x} = \frac{0}{0} \stackrel{\text{BH}}{=} \lim_{x \rightarrow -1} \frac{6x^2 - 2}{9x^2 - 12x - 9} = \frac{4}{12} = \frac{1}{3} \Rightarrow \text{trou en } \left(-1; \frac{1}{3}\right)$$

$$\text{b) } \lim_{x \rightarrow -\infty} \frac{\ln(-x)}{x^2} = \frac{+\infty}{+\infty} \stackrel{\text{BH}}{=} \lim_{x \rightarrow -\infty} \frac{\frac{-1}{-x}}{2x} = \lim_{x \rightarrow -\infty} \frac{1}{2x^2} = \frac{1}{+\infty} = 0 \Rightarrow \text{AHG : } y = 0$$

$$\text{c) } \lim_{x \rightarrow -\infty} \frac{e^{-x}}{x} = \frac{+\infty}{-\infty} \stackrel{\text{BH}}{=} \lim_{x \rightarrow -\infty} \frac{-e^{-x}}{1} = -\infty \Rightarrow \text{aucune AHG}$$

$$\text{d) } \lim_{x \rightarrow +\infty} \frac{e^{3x}}{x+1} = \frac{+\infty}{+\infty} \stackrel{\text{BH}}{=} \lim_{x \rightarrow +\infty} \frac{3e^{3x}}{1} = +\infty \Rightarrow \text{aucune AHD}$$

$$\text{e) } \lim_{x \rightarrow -\infty} \frac{e^x}{x} = \frac{0}{-\infty} = 0 \Rightarrow \text{AHG : } y = 0$$

$$\text{f) } \lim_{x \rightarrow 3} \frac{e^x - 3}{e^x - e^3} = \frac{e^3 - 3}{0} = \pm\infty \Rightarrow \text{AV : } x = 3$$

$$\text{g) } \lim_{x \rightarrow -\infty} \frac{e^{2x} - 1}{e^{3x} + 1} = \frac{-1}{1} = -1 \Rightarrow \text{AHG : } y = -1$$

$$\begin{aligned} \text{h) } \lim_{x \rightarrow +\infty} \frac{e^{2x} - 1}{e^{3x} + 1} &= \frac{+\infty}{+\infty} \stackrel{\text{BH}}{=} \lim_{x \rightarrow +\infty} \frac{2 \cdot e^{2x}}{3 \cdot e^{3x}} \stackrel{\text{BH}}{=} \lim_{x \rightarrow +\infty} \frac{4 \cdot e^{2x}}{9 \cdot e^{3x}} \stackrel{\text{BH}}{=} \lim_{x \rightarrow +\infty} \frac{8 \cdot e^{2x}}{27 \cdot e^{3x}} \stackrel{\text{BH}}{=} \\ &\stackrel{\text{BH}}{=} \lim_{x \rightarrow +\infty} \frac{16 \cdot e^{2x}}{81 \cdot e^{3x}} = \dots = 0 \Rightarrow \text{AHD : } y = 0 \end{aligned}$$