N.E. chapitre 9 : Trigonométrie

Série A — Série B

Exercice 1. (1+1+1=3 pts)

Série A

a) $\sin(135^\circ) = \sin(45^\circ) = \dfrac{\sqrt{2}}{2}$

b) $\tan\left(\dfrac{2\pi}{3}\right) = -\tan\left(\dfrac{\pi}{3}\right) = -\sqrt{3}$

c) $\cos\left(\dfrac{7\pi}{6}\right) = -\cos\left(\dfrac{\pi}{6}\right) = -\dfrac{\sqrt{3}}{2}$

Série B

$\cos(150^\circ) = -\cos(30^\circ) = -\dfrac{\sqrt{3}}{2}$

$\tan\left(\dfrac{4\pi}{3}\right) = \tan\left(\dfrac{\pi}{3}\right) = \sqrt{3}$

$\sin\left(\dfrac{5\pi}{4}\right) = -\sin\left(\dfrac{\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}$

Exercice 2. (2+5=7 pts)

Série A

a) $\tan\left(\dfrac{t}{2} - \dfrac{\pi}{3}\right) = \tan\left(\dfrac{\pi}{3} + k\cdot\pi\right), k \in \mathbb{Z}$

$\Rightarrow \dfrac{t}{2} = \dfrac{2\pi}{3} + k\cdot\pi, k \in \mathbb{Z}$

$\Rightarrow t = \left\{\dfrac{4\pi}{3} + k\cdot 2\pi, k \in \mathbb{Z}\right\}$

b) $\begin{cases} \sin(t) + \cos(t) = 1 \\ \sin^2(t) + \cos^2(t) = 1 \end{cases}$

$\Rightarrow \begin{cases} \cos(t) = 1 - \sin(t) \\ 2\sin^2(t) - 2\sin(t) = 0 \end{cases}$

$\Rightarrow \begin{cases} \cos(t) = 1 - \sin(t) \\ 2\sin(t)[\sin(t) - 1] = 0 \end{cases}$

$\Rightarrow \begin{cases} \cos(t) = 1 \\ \sin(t) = 0 \end{cases}$ ou $\begin{cases} \cos(t) = 0 \\ \sin(t) = 1 \end{cases}$

$\Rightarrow t_1 = \{k\cdot 2\pi, k \in \mathbb{Z}\}$

$t_2 = \left\{\dfrac{\pi}{2} + k\cdot 2\pi, k \in \mathbb{Z}\right\}$

Série B

$\tan\left(\dfrac{t}{4} + \dfrac{\pi}{3}\right) = \tan\left(\dfrac{2\pi}{3} + k\cdot\pi\right), k \in \mathbb{Z}$

$\Rightarrow \dfrac{t}{4} = \dfrac{\pi}{3} + k\cdot\pi, k \in \mathbb{Z}$

$\Rightarrow t = \left\{\dfrac{4\pi}{3} + k\cdot 4\pi, k \in \mathbb{Z}\right\}$

$\begin{cases} \sin(t) - \cos(t) = 1 \\ \sin^2(t) + \cos^2(t) = 1 \end{cases}$

$\Rightarrow \begin{cases} \cos(t) = \sin(t) - 1 \\ 2\sin^2(t) - 2\sin(t) = 0 \end{cases}$

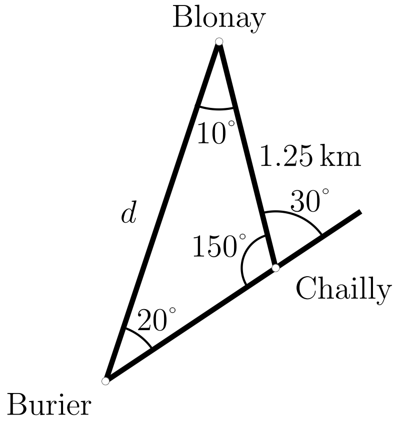
$\Rightarrow \begin{cases} \cos(t) = \sin(t) - 1 \\ 2\sin(t)[\sin(t) - 1] = 0 \end{cases}$

$\Rightarrow \begin{cases} \cos(t) = -1 \\ \sin(t) = 0 \end{cases}$ ou $\begin{cases} \cos(t) = 0 \\ \sin(t) = 1 \end{cases}$

$\Rightarrow t_1 = \{\pi + k\cdot 2\pi, k \in \mathbb{Z}\}$

$t_2 = \left\{\dfrac{\pi}{2} + k\cdot 2\pi, k \in \mathbb{Z}\right\}$

Exercice 3. (5 pts)



• La distance $Ch - Bl = 5 \cdot \frac{1}{4} = 1.25\,\text{km}$

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• Un côté et deux angles sont connus ⇒ théorème du sinus : $\dfrac{d}{\sin(150^\circ)} = \dfrac{1.25}{\sin(20^\circ)}$

$$\Rightarrow \quad d = \frac{1.25 \cdot \sin(150^\circ)}{\sin(20^\circ)} \cong 1.827\,\text{km}$$

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• La distance entre le parking supérieur du gymnase de Burier et Blonay est d'environ 1.827 km

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Exercice 4. (5 pts)

$$\begin{cases} 3\sin^2(t) - 2\cos(t) - 2 = 0 \\ \sin^2(t) + \cos^2(t) = 1 \end{cases}$$

$$\Rightarrow \begin{cases} 3\cos^2(t) + 2\cos(t) - 1 = 0 \\ \sin^2(t) = 1 - \cos^2(t) \end{cases}$$

$$x = \cos(t) \Rightarrow 3x^2 + 2x - 1 = 0$$

$$\iff (3x - 1)(x + 1) = 0$$

$$\Rightarrow \cos(t) = -1 \text{ ou } \cos(t) = \frac{1}{3}$$

$$\Rightarrow t_1 = \{\pi + k \cdot 2\pi, k \in \mathbb{Z}\}$$

$$t_2 = \{\arccos(1/3) + k \cdot 2\pi, k \in \mathbb{Z}\}$$

$$t_3 = \{-\arccos(1/3) + k \cdot 2\pi, k \in \mathbb{Z}\}$$

$$\begin{cases} 3\sin^2(t) + 2\cos(t) - 2 = 0 \\ \sin^2(t) + \cos^2(t) = 1 \end{cases}$$

$$\Rightarrow \begin{cases} 3\cos^2(t) - 2\cos(t) - 1 = 0 \\ \sin^2(t) = 1 - \cos^2(t) \end{cases}$$

$$x = \cos(t) \Rightarrow 3x^2 - 2x - 1 = 0$$

$$\iff (3x + 1)(x - 1) = 0$$

$$\Rightarrow \cos(t) = 1 \text{ ou } \cos(t) = -\frac{1}{3}$$

$$\Rightarrow t_1 = \{k \cdot 2\pi, k \in \mathbb{Z}\}$$

$$t_2 = \{\arccos(-1/3) + k \cdot 2\pi, k \in \mathbb{Z}\}$$

$$t_3 = \{-\arccos(-1/3) + k \cdot 2\pi, k \in \mathbb{Z}\}$$
