

N.E. ch.6 : Fonctions puissances, exponentielles et logarithmesSérie ASérie B**Exercice 1.** (4 pts)

a) $(a^3 \cdot a^{-5})^2 = (a^{-2})^2 = a^{-4}$

b) $\sqrt[4]{a^3} \cdot \sqrt[4]{a^5} = \sqrt[4]{a^8} = a^2$

c) $\log_a \left(a^n \cdot \frac{1}{a^{n+1}} \right) = \log_a(a^{-1}) = -1$

d) $\log_a \left(\sqrt[3]{a^2} \right)^4 = \log_a \left(a^{\frac{8}{3}} \right) = \frac{8}{3}$

$(a^2 \cdot a^{-4})^3 = (a^{-2})^3 = a^{-6}$

$\sqrt[4]{a^2} \cdot \sqrt[4]{a^{10}} = \sqrt[4]{a^{12}} = a^3$

$\log_a \left(a^n \cdot \frac{1}{a^{n+2}} \right) = \log_a(a^{-2}) = -2$

$\log_a \left(\sqrt[3]{a^4} \right)^2 = \log_a \left(a^{\frac{8}{3}} \right) = \frac{8}{3}$

Exercice 2. (3 pts)

1) VAR : $t = \text{durée en h } (t \geq 0)$

2) EQ : $\frac{q_0}{3} = q_0 \cdot 0.8^t \quad | : q_0 \neq 0$

3) RES : $\Rightarrow \frac{1}{3} = 0.8^t \Rightarrow \log(1/3) = \log(0.8^t)$

$$\Rightarrow t = \frac{\log(1/3)}{\log(0.8)} (\cong 4.92 \text{ h})$$

4) SOL : ... après environ 4 h et 55 min.

VAR : $t = \text{durée en h } (t \geq 0)$

EQ : $\frac{q_0}{2} = q_0 \cdot 0.9^t \quad | : q_0 \neq 0$

RES : $\Rightarrow \frac{1}{2} = 0.9^t \Rightarrow \log(1/2) = \log(0.9^t)$

$$\Rightarrow t = \frac{\log(1/2)}{\log(0.9)} (\cong 6.58 \text{ h})$$

SOL : ... après environ 6 h et 35 min.

Exercice 3. (1.5+1.5+3+3=9 pts)

a) $3^{x+1} \cdot 9^{-x} = 81 \Rightarrow 3^{x+1} \cdot 3^{-2x} = 3^4 \Rightarrow$

$$\Rightarrow x + 1 - 2x = 4 \Rightarrow x = -3$$

b) $4 \cdot e^{3x} = 2\pi \Rightarrow x = \frac{\ln(\pi/2)}{3} (\cong 0.151)$

c) $2^x = 3^{x-3} \Rightarrow x \cdot \log(2) = (x-3) \log(3) \Rightarrow$

$$\Rightarrow x \cdot \log(3) - x \cdot \log(2) = 3 \log(3) \Rightarrow$$

$$\Rightarrow x[\log(3) - \log(2)] = 3 \log(3) \Rightarrow$$

$$\Rightarrow x = \frac{3 \log(3)}{\log(\frac{3}{2})} (\cong 8.129)$$

d) $ED = \mathbb{R}_+^*$

$$\log(10x + 9) = \log(x) + \log(100) \Rightarrow$$

$$\Rightarrow \log(10x + 9) = \log(100x) \Rightarrow$$

$$\Rightarrow 10x + 9 = 100x \Rightarrow x = \frac{1}{10} \in \mathbb{R}_+^*$$

$$2^{x+3} \cdot 8^{-x} = 32 \Rightarrow 2^{x+3} \cdot 2^{-3x} = 2^5 \Rightarrow$$

$$\Rightarrow x + 3 - 3x = 5 \Rightarrow x = -1$$

$$6 \cdot e^{2x} = 3\pi \Rightarrow x = \frac{\ln(\pi/2)}{2} (\cong 0.226)$$

$$3^x = 2^{x-2} \Rightarrow x \cdot \log(3) = (x-2) \log(2) \Rightarrow$$

$$\Rightarrow x \cdot \log(3) - x \cdot \log(2) = -2 \log(2) \Rightarrow$$

$$\Rightarrow x[\log(3) - \log(2)] = -2 \log(2) \Rightarrow$$

$$\Rightarrow x = -\frac{2 \log(2)}{\log(\frac{3}{2})} (\cong -3.42)$$

$$ED = \mathbb{R}_+^*$$

$$\log(30x + 7) = \log(x) + \log(100) \Rightarrow$$

$$\Rightarrow \log(30x + 7) = \log(100x) \Rightarrow$$

$$\Rightarrow 30x + 7 = 100x \Rightarrow x = \frac{1}{10} \in \mathbb{R}_+^*$$

Exercice 4. (1+3=4 pts)

a) $p(2) = 3 + 20(1 - e^{-0.2}) \cong 6.6$ articles.

b) VAR : n désigne le $n^{\text{ème}}$ jour de travail

INEQ : $3 + 20(1 - e^{-0.1n}) > 20$

RES : $\Rightarrow 20(1 - e^{-0.1n}) > 17$

$$\Rightarrow 1 - e^{-0.1n} > \frac{17}{20}$$

$$\Rightarrow e^{-0.1n} < \frac{3}{20}$$

$$\Rightarrow -0.1 \cdot n < \ln(3/20)$$

$$\Rightarrow n > 10 \cdot \ln(20/3) (\cong 18.97 \text{ j})$$

SOL : ... plus de 20 art. durant le 19^{ème} jour.

$$p(3) = 7 + 10(1 - e^{-0.6}) \cong 11.5$$
 articles.

VAR : n désigne le $n^{\text{ème}}$ jour de travail

INEQ : $7 + 10(1 - e^{-0.2n}) > 15$

RES : $\Rightarrow 10(1 - e^{-0.2n}) > 8$

$$\Rightarrow 1 - e^{-0.2n} > \frac{8}{10}$$

$$\Rightarrow e^{-0.2n} < \frac{1}{5}$$

$$\Rightarrow -0.2 \cdot n < \ln(1/5)$$

$$\Rightarrow n > 5 \cdot \ln(5) (\cong 8.05 \text{ j})$$

SOL : ... plus de 15 art. durant le 9^{ème} jour.