Analyse - §2 : limites, continuité et asymptotes

Exercice 1. (1.5+1.5+2+1=6 pts)

Série A

a) $\lim_{x\to -1} \frac{x^2+x-1}{x^2+2x+1} = \lim_{x\to -1} \frac{x^2+x-1}{(x+1)^2} =$ „$\frac{-1}{0_+}$" $= -\infty$

b) $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} = \lim_{x\to 9} \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{(x-9)(\sqrt{x}+3)} = \lim_{x\to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{6}$

c) $\lim_{x\to 0} \frac{\sin(x)}{x^2-2x} = \lim_{x\to 0} \frac{\sin(x)}{x}\cdot\frac{1}{x-2} = 1\cdot\frac{1}{-2} = -\frac{1}{2}$

d) $\lim_{x\to -\infty} \frac{3x^2-2x+1}{x^3+9x^2-73x+99} = \lim_{x\to -\infty} \frac{3x^2}{x^3} = \lim_{x\to -\infty} \frac{3}{x} = 0$

Série B

a) $\lim_{x\to -1} \frac{x^2+x+1}{x^2+2x+1} = \lim_{x\to -1} \frac{x^2+x+1}{(x+1)^2} =$ „$\frac{1}{0_+}$" $= +\infty$

b) $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4} = \lim_{x\to 4} \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{(x-4)(\sqrt{x}+2)} = \lim_{x\to 4} \frac{1}{\sqrt{x}+2} = \frac{1}{4}$

c) $\lim_{x\to 0} \frac{\sin(x)}{x^2-5x} = \lim_{x\to 0} \frac{\sin(x)}{x}\cdot\frac{1}{x-5} = 1\cdot\frac{1}{-5} = -\frac{1}{5}$

d) $\lim_{x\to -\infty} \frac{4x^2-3x+2}{x^3+7x^2-65x+98} = \lim_{x\to -\infty} \frac{4x^2}{x^3} = \lim_{x\to -\infty} \frac{4}{x} = 0$

Exercice 2. (3 pts)

Série A

- AV : $x=-1$, $x=2$
- AH : $y=2$
- $Z_f = \{0\,;3\}$

par exemple : $f(x) = \frac{2x(x-3)}{(x+1)(x-2)}$

Série B

- AV : $x=-1$, $x=2$
- AH : $y=-2$
- $Z_f = \{0\,;3\}$

par exemple : $f(x) = -\frac{2x(x-3)}{(x+1)(x-2)}$

Exercice 3. (1+1+5+2=9 pts)

a) $g(x) = \dfrac{x(x-1)(x+1)}{(x-1)(x-2)} \Rightarrow ED_g = \mathbb{R} \setminus \{1\,;2\}$

b) $Z_g = \{-1\,;0\}$

x		-1		0		1		2	
sgn(g)	$-$	0	$+$	0	$-$	‖	$-$	‖	$+$

c) $\lim\limits_{x\to 1} g(x) = \lim\limits_{x\to 1} \dfrac{x(x+1)}{(x-2)} = -2 \Rightarrow$

$\Rightarrow$ **trou en** $(1\,;-2)$

$\lim\limits_{\substack{x\to 2\\<}} g(x) = \lim\limits_{\substack{x\to 2\\<}} \dfrac{x(x+1)}{(x-2)} = „\dfrac{6}{0_-}“ = -\infty$

$\lim\limits_{\substack{x\to 2\\>}} g(x) = \lim\limits_{\substack{x\to 2\\>}} \dfrac{x(x+1)}{(x-2)} = „\dfrac{6}{0_+}“ = +\infty \Rightarrow$

$\Rightarrow$ **asymptote verticale en** $x = 2$

$\lim\limits_{x\to\pm\infty} g(x) = \lim\limits_{x\to\pm\infty} \dfrac{x^3}{x^2} = \pm\infty \Rightarrow$

$\Rightarrow$ **pas d'asymptote horizontale**

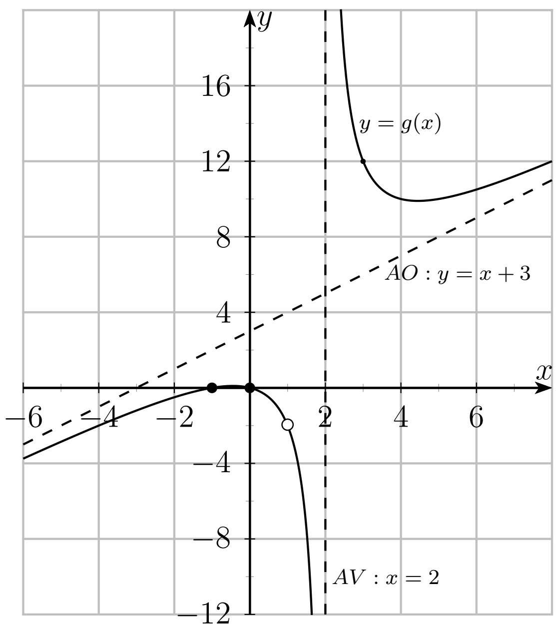
div. eucli. : $g(x) = x + 3 + \dfrac{6x-6}{(x-1)(x-2)} \Rightarrow$

$\Rightarrow$ **asymptote oblique en** $y = x + 3$

d)

x		1		2	
sgn(δ)	$-$	‖	$-$	‖	$+$
position	sous	‖	sous	‖	sur

e)



$g(x) = \dfrac{x(x-1)(x+1)}{(x-1)(x-3)} \Rightarrow ED_g = \mathbb{R} \setminus \{1\,;3\}$

$Z_g = \{-1\,;0\}$

x		-1		0		1		3	
sgn(g)	$-$	0	$+$	0	$-$	‖	$-$	‖	$+$

$\lim\limits_{x\to 1} g(x) = \lim\limits_{x\to 1} \dfrac{x(x+1)}{(x-3)} = -1 \Rightarrow$

$\Rightarrow$ **trou en** $(1\,;-1)$

$\lim\limits_{\substack{x\to 3\\<}} g(x) = \lim\limits_{\substack{x\to 3\\<}} \dfrac{x(x+1)}{(x-3)} = „\dfrac{12}{0_-}“ = -\infty$

$\lim\limits_{\substack{x\to 3\\>}} g(x) = \lim\limits_{\substack{x\to 3\\>}} \dfrac{x(x+1)}{(x-2)} = „\dfrac{12}{0_+}“ = +\infty \Rightarrow$

$\Rightarrow$ **asymptote verticale en** $x = 3$

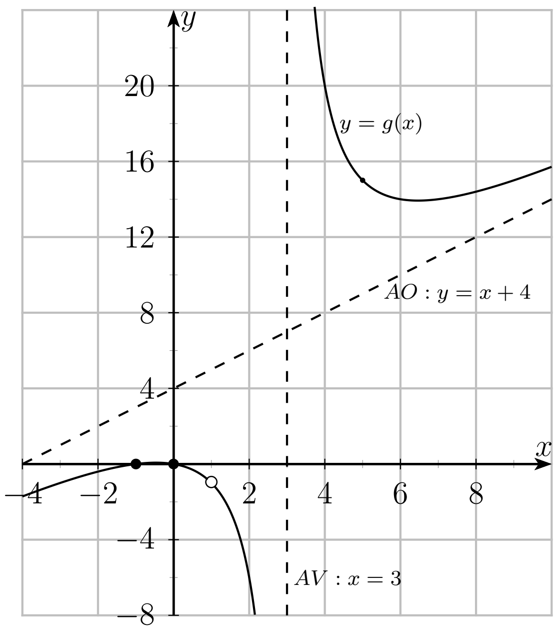
$\lim\limits_{x\to\pm\infty} g(x) = \lim\limits_{x\to\pm\infty} \dfrac{x^3}{x^2} = \pm\infty \Rightarrow$

$\Rightarrow$ **pas d'asymptote horizontale**

div. eucli. : $g(x) = x + 4 + \dfrac{12x-12}{(x-1)(x-3)} \Rightarrow$

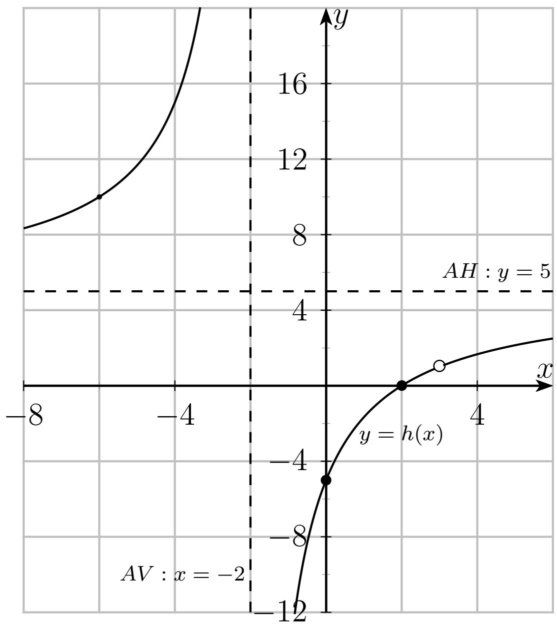
$\Rightarrow$ **asymptote oblique en** $y = x + 4$

x		1		3	
sgn(δ)	$-$	‖	$-$	‖	$+$
position	sous	‖	sous	‖	sur



Exercice 4. (2 pts)

par exemple : $h(x) = \dfrac{5(x-2)(x-3)}{(x+2)(x-3)}$



par exemple : $h(x) = -\dfrac{5(x-2)(x-3)}{(x+2)(x-3)}$