Exercice 1.

Il s'agit de montrer que la norme de chacun de ces vecteurs vaut 1.

Exercice 2.

a) sans corrigé.

b) • On cherche un vecteur $\overrightarrow{v_u}$ tel que $\overrightarrow{v_u} = k \cdot \overrightarrow{a}$ et $||\overrightarrow{v_u}|| = 1\,[\text{u}]$

• Méthode 1 :

$||\overrightarrow{v_u}|| = ||k \cdot \overrightarrow{a}|| = ||k \cdot \begin{pmatrix} 3 \\ 4 \end{pmatrix}|| = ||\begin{pmatrix} 3k \\ 4k \end{pmatrix}|| = \sqrt{(3k)^2 + (4k)^2} = \sqrt{25k^2} = 1 \iff$

$\iff 25k^2 = 1 \iff 25k^2 - 1 = 0 \iff (5k+1)(5k-1) = 0 \iff k_1 = -\frac{1}{5}$ ou $k_2 = \frac{1}{5}$

$\Rightarrow \boxed{\overrightarrow{v_u} = \pm\frac{1}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix}}$

• Méthode 2 :

$\overrightarrow{a_u} = \frac{\overrightarrow{a}}{||\overrightarrow{a}||} = \begin{pmatrix} 3/5 \\ 4/5 \end{pmatrix} \Rightarrow \overrightarrow{v_u} = \pm\overrightarrow{a_u} \iff \overrightarrow{v_u} = \pm\begin{pmatrix} 3/5 \\ 4/5 \end{pmatrix} \iff \boxed{\overrightarrow{v_u} = \pm\frac{1}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix}}$

Idem avec les autres vecteurs.

Exercice 3.

a) sans corrigé.

b) • On cherche un vecteur $\overrightarrow{v_u}$ tel que $\overrightarrow{v_u} = k \cdot \overrightarrow{a}$ et $||\overrightarrow{v_u}|| = 1\,[\text{u}]$

• Méthode 1 :

$||\overrightarrow{v_u}|| = ||k \cdot \overrightarrow{a}|| = ||k \cdot \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}|| = ||\begin{pmatrix} k \\ 2k \\ -2k \end{pmatrix}|| = \sqrt{k^2 + (2k)^2 + (-2k)^2} = \sqrt{9k^2} = 1$

$\iff 9k^2 = 1 \iff 9k^2 - 1 = 0 \iff (3k+1)(3k-1) = 0 \iff k_1 = -\frac{1}{3}$ ou $k_2 = \frac{1}{3}$

On veut un vecteur de sens contraire, on ne garde que $k_1 = -\frac{1}{3} \Rightarrow \boxed{\overrightarrow{v_u} = -\frac{1}{3}\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}}$

• Méthode 2 :

$\overrightarrow{a_u} = \frac{\overrightarrow{a}}{||\overrightarrow{a}||} = \begin{pmatrix} 1/3 \\ 2/3 \\ -2/3 \end{pmatrix} \Rightarrow \overrightarrow{v_u} = -\overrightarrow{a_u} \iff \overrightarrow{v_u} = -\begin{pmatrix} 1/3 \\ 2/3 \\ -2/3 \end{pmatrix} \iff \boxed{\overrightarrow{v_u} = -\frac{1}{3}\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}}$

Idem avec les autres vecteurs.

Exercice 4.

a) • On cherche un vecteur $\overrightarrow{v_u}$ tel que $\overrightarrow{v_u} = k \cdot \overrightarrow{a}$ et $||\overrightarrow{v_u}|| = 1\,[\text{u}]$

• Méthode 1 :

$$||\overrightarrow{v_u}|| = ||k \cdot \overrightarrow{a}|| = ||k \cdot \begin{pmatrix} 2 \\ -7 \end{pmatrix}|| = ||\begin{pmatrix} 2k \\ -7k \end{pmatrix}|| = \sqrt{(2k)^2 + (-7k)^2} = \sqrt{53k^2} = 1 \iff$$

$$\iff 53k^2 = 1 \iff 53k^2 - 1 = 0 \iff (\sqrt{53}\,k + 1)(\sqrt{53}\,k - 1) = 0 \iff$$

$$\iff k_1 = -\frac{1}{\sqrt{53}} \text{ ou } k_2 = \frac{1}{\sqrt{53}} \Rightarrow \boxed{\overrightarrow{v_u} = \pm\frac{1}{\sqrt{53}}\begin{pmatrix} 2 \\ -7 \end{pmatrix}}$$

• Méthode 2 :

$$\overrightarrow{a_u} = \frac{\overrightarrow{a}}{||\overrightarrow{a}||} = \begin{pmatrix} \frac{2}{\sqrt{53}} \\ -\frac{7}{\sqrt{53}} \end{pmatrix} \Rightarrow \overrightarrow{v_u} = \pm\overrightarrow{a_u} \iff \overrightarrow{v_u} = \pm\begin{pmatrix} \frac{2}{\sqrt{53}} \\ -\frac{7}{\sqrt{53}} \end{pmatrix} \iff \boxed{\overrightarrow{v_u} = \pm\frac{1}{\sqrt{53}}\begin{pmatrix} 2 \\ -7 \end{pmatrix}}$$

b) Idem partie a, mais en multipliant $\frac{1}{\sqrt{53}}$ par 5.

Exercice 5.

a) • On cherche un vecteur $\overrightarrow{v_u}$ tel que $\overrightarrow{v_u} = k \cdot \overrightarrow{a}$ et $||\overrightarrow{v_u}|| = 1\,[\text{u}]$

• Méthode 1 :

$$||\overrightarrow{v_u}|| = ||k \cdot \overrightarrow{a}|| = ||k \cdot \begin{pmatrix} 14 \\ -8 \\ 8 \end{pmatrix}|| = ||\begin{pmatrix} 14k \\ -8k \\ 8k \end{pmatrix}|| = \sqrt{(14k)^2 + (-8k)^2 + (8k)^2} =$$

$$= \sqrt{324k^2} = 1 \iff 324k^2 = 1 \iff 324k^2 - 1 = 0 \iff (18k+1)(18k-1) = 0 \iff$$

$$\iff k_1 = -\frac{1}{18} \text{ ou } k_2 = \frac{1}{18} \Rightarrow \overrightarrow{v_u} = \pm\frac{1}{18}\begin{pmatrix} 14 \\ -8 \\ 8 \end{pmatrix} \Rightarrow \boxed{\overrightarrow{v_u} = \pm\frac{1}{9}\begin{pmatrix} 7 \\ -4 \\ 4 \end{pmatrix}}$$

• Méthode 2 :

$$\overrightarrow{a_u} = \frac{\overrightarrow{a}}{||\overrightarrow{a}||} = \begin{pmatrix} 14/18 \\ -8/18 \\ 8/18 \end{pmatrix} \Rightarrow \overrightarrow{v_u} = \pm\overrightarrow{a_u} \iff \overrightarrow{v_u} = \pm\begin{pmatrix} 7/9 \\ -4/9 \\ 4/9 \end{pmatrix} \iff \boxed{\overrightarrow{v_u} = \pm\frac{1}{9}\begin{pmatrix} 7 \\ -4 \\ 4 \end{pmatrix}}$$

b) Idem partie a, mais en multipliant $\frac{1}{9}$ par 9.

Exercice 6.

La distance entre les points A et $B = \delta(A\,;B) = ||\overrightarrow{AB}||$ (voir théorie p.52)

a) • $\overrightarrow{AB} = \begin{pmatrix} 3-5 \\ 0-(-3) \\ 9-1 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ 8 \end{pmatrix}$

• $||\overrightarrow{AB}|| = \sqrt{(-2)^2+3^2+8^2} = \boxed{\sqrt{77}\,[\text{u}]}$

b) • $\overrightarrow{AB} = \begin{pmatrix} 10 \\ -6 \\ -15 \end{pmatrix}$

• $||\overrightarrow{AB}|| = \sqrt{10^2+(-6)^2+(-15)^2} = \sqrt{361} = \boxed{19\,[\text{u}]}$

c) • $\overrightarrow{AB} = \begin{pmatrix} 5 \\ 6 \\ 6 \end{pmatrix}$

• $||\overrightarrow{AB}|| = \sqrt{5^2+6^2+6^2} = \boxed{\sqrt{97}\,[\text{u}]}$

d) • $\overrightarrow{AB} = \begin{pmatrix} 8 \\ -3 \\ 9 \end{pmatrix}$

• $||\overrightarrow{AB}|| = \sqrt{8^2+(-3)^2+9^2} = \boxed{\sqrt{154}\,[\text{u}]}$

Si vous avez tout corrigé ces exercices, envoyez-moi un mail de confirmation avec d'éventuelles questions.

Exercice 7.

a) • $\overrightarrow{AB} = \begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix}$; $\overrightarrow{BC} = \begin{pmatrix} -2 \\ -3 \\ 6 \end{pmatrix}$; $\overrightarrow{CD} = \begin{pmatrix} -6 \\ -2 \\ -3 \end{pmatrix}$; $\overrightarrow{AD} = \begin{pmatrix} -2 \\ -3 \\ 6 \end{pmatrix}$

$\Rightarrow \overrightarrow{AB} = \overrightarrow{DC}$; $\overrightarrow{BC} = \overrightarrow{AD}$

• $||\overrightarrow{AB}|| = ||\overrightarrow{BC}|| = ||\overrightarrow{CD}|| = ||\overrightarrow{AD}|| = \sqrt{49}\,[\text{u}] = 7\,[\text{u}]$

• $\overrightarrow{AC} = \begin{pmatrix} 4 \\ -1 \\ 9 \end{pmatrix}$; $\overrightarrow{BD} = \begin{pmatrix} -8 \\ -5 \\ 3 \end{pmatrix} \Rightarrow ||\overrightarrow{AC}|| = ||\overrightarrow{BD}|| = \sqrt{98} = \sqrt{49 \cdot 2} = 7\sqrt{2}\,[\text{u}]$

$\Rightarrow$ $ABCD$ est un quadrilatère avec 4 côtés isométriques et les diagonales isométriques, donc c'est un losange particulier : un carré.

b) • $\overrightarrow{AB} = \begin{pmatrix} 7 \\ 1 \\ -4 \end{pmatrix}$; $\overrightarrow{BC} = \begin{pmatrix} -5 \\ 4 \\ 5 \end{pmatrix}$; $\overrightarrow{CD} = \begin{pmatrix} -7 \\ -1 \\ 4 \end{pmatrix}$; $\overrightarrow{AD} = \begin{pmatrix} -5 \\ 4 \\ 5 \end{pmatrix}$

$\Rightarrow \overrightarrow{AB} = \overrightarrow{DC}$; $\overrightarrow{BC} = \overrightarrow{AD}$

• $||\overrightarrow{AB}|| = ||\overrightarrow{BC}|| = ||\overrightarrow{CD}|| = ||\overrightarrow{AD}|| = \sqrt{66}\,[\text{u}]$

• $\overrightarrow{AC} = \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix}$; $\overrightarrow{BD} = \begin{pmatrix} -12 \\ 3 \\ 9 \end{pmatrix} \Rightarrow ||\overrightarrow{AC}|| = \sqrt{30}\,[\text{u}]$; $||\overrightarrow{BD}|| = \sqrt{234} = 3\sqrt{26}\,[\text{u}]$

$\Rightarrow$ $ABCD$ est un quadrilatère avec 4 côtés isométriques et les diagonales non isométriques, donc c'est un losange.

Exercice 8.

• On pose $P(p_1; p_2)$

• $\overrightarrow{AB_u} = \dfrac{\overrightarrow{AB}}{||\overrightarrow{AB}||} = \dfrac{1}{15}\begin{pmatrix} -9 \\ 12 \end{pmatrix} = \begin{pmatrix} -9/15 \\ 12/15 \end{pmatrix} = \begin{pmatrix} -3/5 \\ 4/5 \end{pmatrix}$

• $\overrightarrow{AP} = \pm 3 \cdot \overrightarrow{AB_u} \Rightarrow \begin{pmatrix} p_1 - 4 \\ p_2 + 1 \end{pmatrix} = \pm 3 \cdot \begin{pmatrix} -3/5 \\ 4/5 \end{pmatrix} = \pm \begin{pmatrix} -9/5 \\ 12/5 \end{pmatrix}$

$+ \Rightarrow \boxed{P_1\left(\dfrac{11}{5} ; \dfrac{7}{5}\right)}$

$- \Rightarrow \boxed{P_2\left(\dfrac{29}{5} ; -\dfrac{17}{5}\right)}$

Exercice 9.

- On cherche $k \in \mathbb{R}$ tel que $||\overrightarrow{a} + k \cdot \overrightarrow{b}|| = \sqrt{82}$ [u]

- $\overrightarrow{a} + k \cdot \overrightarrow{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + k \begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} 2-2k \\ 3+4k \end{pmatrix}$

- $||\overrightarrow{a} + k \cdot \overrightarrow{b}|| = \sqrt{(2-2k)^2 + (3+4k)^2} = \sqrt{4 - 8k + 4k^2 + 9 + 24k + 16k^2} =$

$= \sqrt{20k^2 + 16k + 13} = \sqrt{82} \iff 20k^2 + 16k + 13 = 82 \iff 20k^2 + 16k - 69 = 0 \iff$

$\iff (10k+23)(2k-3) = 0 \iff \boxed{k_1 = -\frac{23}{10} \text{ ou } k_2 = \frac{3}{2}}$

Exercice 10.

- $\overrightarrow{AB} = \begin{pmatrix} -9 \\ 9 \end{pmatrix}$; $\overrightarrow{AC} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$; $\overrightarrow{BC} = \begin{pmatrix} 9 \\ -10 \end{pmatrix}$

- $||\overrightarrow{AB}|| = \sqrt{162} = \sqrt{81 \cdot 2} = \sqrt{81} \cdot \sqrt{2} = 9\sqrt{2}$ [u]

- $||\overrightarrow{AC}|| = 1$ [u]

- $||\overrightarrow{BC}|| = \sqrt{181}$ [u]

Exercice 12.

- $\overrightarrow{AB} = \begin{pmatrix} -10/9 \\ 40/9 \\ -80/9 \end{pmatrix} \Rightarrow ||\overrightarrow{AB}|| = \sqrt{\frac{8100}{81}} = \sqrt{100} = 10\,[\text{u}]$

- $\overrightarrow{BC} = \begin{pmatrix} (-20\sqrt{2}+45)/9 \\ 35\sqrt{2}/9 \\ (20\sqrt{2}+45)/9 \end{pmatrix} \Rightarrow$

$\Rightarrow ||\overrightarrow{BC}|| = \sqrt{\frac{(800-1800\sqrt{2}+2025)+2450+(800+1800\sqrt{2}+2025)}{81}} = \sqrt{\frac{8100}{81}} = 10\,[\text{u}]$

- $\overrightarrow{AC} = \begin{pmatrix} (-20\sqrt{2}+35)/9 \\ (35\sqrt{2}+40)/9 \\ (20\sqrt{2}-35)/9 \end{pmatrix} \Rightarrow$

$\Rightarrow ||\overrightarrow{AC}|| = \sqrt{\frac{(800-1400\sqrt{2}+1225)+(2450+2800\sqrt{2}+1600)+(800-1400\sqrt{2}+1225)}{81}} = \sqrt{\frac{8100}{81}} = 10\,[\text{u}]$

$\Rightarrow$ ABC est un triangle avec 3 côtés isométriques, donc c'est un triangle équilatéral.

Exercice 13.

- $\overrightarrow{AI} = \begin{pmatrix} -5 \\ 0 \end{pmatrix} \Rightarrow ||\overrightarrow{AI}|| = \sqrt{25} = 5\,[\text{u}]$

- $\overrightarrow{BI} = \begin{pmatrix} -3 \\ -4 \end{pmatrix} \Rightarrow ||\overrightarrow{BI}|| = \sqrt{25} = 5\,[\text{u}]$

- $\overrightarrow{CI} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} \Rightarrow ||\overrightarrow{CI}|| = \sqrt{25} = 5\,[\text{u}]$

$\Rightarrow ||\overrightarrow{AI}|| = ||\overrightarrow{BI}|| = ||\overrightarrow{CI}|| = 5\,[\text{u}]$

I est donc équidistant à A, B et C donc I est le centre du cercle de rayon égal à $5\,[\text{u}]$

Exercice 16.

• On pose $P(p_1;p_2)$

• $\overrightarrow{AB}_u = \dfrac{\overrightarrow{AB}}{||\overrightarrow{AB}||} = \dfrac{1}{13}\begin{pmatrix}12\\-5\end{pmatrix} = \begin{pmatrix}12/13\\-5/13\end{pmatrix}$

• $\overrightarrow{AP} = 7\cdot\overrightarrow{AB}_u \Rightarrow \begin{pmatrix}p_1+2\\p_2-3\end{pmatrix} = 7\cdot\begin{pmatrix}12/13\\-5/13\end{pmatrix} = \begin{pmatrix}84/13\\-35/13\end{pmatrix}$

$\Rightarrow \boxed{P\left(\frac{58}{13};\frac{4}{13}\right)}$

Exercice 17.

• $P\in Ox \Rightarrow P(x\,;0\,;0)$

• $\overrightarrow{AP} = \begin{pmatrix}x-4\\-5\\-8\end{pmatrix} \quad ; \quad \overrightarrow{BP} = \begin{pmatrix}x-3\\-11\\-5\end{pmatrix}$

a) $||\overrightarrow{AP}|| = ||\overrightarrow{BP}|| \iff$

$\iff \sqrt{(x-4)^2+(-5)^2+(-8)^2} = \sqrt{(x-3)^2+(-11)^2+(-5)^2} \overset{()^2}{\Rightarrow}$

$\overset{()^2}{\Rightarrow} (x-4)^2+(-5)^2+(-8)^2 = (x-3)^2+(-11)^2+(-5)^2 \iff$

$\iff x^2-8x+16+25+64 = x^2-6x+9+121+25 \iff$

$\iff 2x=-50 \iff x=-25$

$\Rightarrow \boxed{P(-25\,;0\,;0)}$

b) $||\overrightarrow{AP}|| = 2\cdot||\overrightarrow{BP}|| \iff$

$\iff \sqrt{(x-4)^2+(-5)^2+(-8)^2} = 2\cdot\sqrt{(x-3)^2+(-11)^2+(-5)^2} \overset{()^2}{\Rightarrow}$

$\overset{()^2}{\Rightarrow} x^2-8x+105 = 4[x^2-6x+155] \iff x^2-8x+105 = 4x^2-24x+620 \iff$

$\iff 3x^2-16x+515=0 \quad ; \quad \Delta = (-16)^2-4\cdot3\cdot515 = 256-6180 < 0$

$\Rightarrow \boxed{\text{pas de solution}}$

Exercice 19.

$* \vec{a} \bullet \vec{b} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} -1 \\ -5 \end{pmatrix} = 3 \cdot (-1) + 1 \cdot (-5) = -3 - 5 = \boxed{-8}$

$* \vec{a} \bullet \vec{c} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -3 \end{pmatrix} = 3 \cdot 1 + 1 \cdot (-3) = 3 - 3 = \boxed{0} \iff \vec{a} \perp \vec{c}$

$* \vec{a} \bullet \vec{e} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} -4 \\ 3 \end{pmatrix} = 3 \cdot (-4) + 1 \cdot 3 = -12 + 3 = \boxed{-9}$

$* \vec{b} \bullet \vec{a} = \begin{pmatrix} -1 \\ -5 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 1 \end{pmatrix} = (-1) \cdot 3 + (-5) \cdot 1 = -3 - 5 = \boxed{-8} = \vec{a} \bullet \vec{b}$

$* \vec{d} \bullet \vec{e} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} -4 \\ 3 \end{pmatrix} = 3 \cdot (-4) + 4 \cdot 3 = -12 + 12 = \boxed{0} \iff \vec{d} \perp \vec{e}$

$* \vec{c} \bullet \vec{d} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 4 \end{pmatrix} = 1 \cdot 3 + (-3) \cdot 4 = 3 - 12 = \boxed{-9}$

$* (\vec{a} + \vec{b}) \bullet (\vec{c} - \vec{d}) = \begin{pmatrix} 2 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} -2 \\ -7 \end{pmatrix} = 2 \cdot (-2) + (-4) \cdot (-7) = -4 + 28 = \boxed{24}$

Exercice 20.

$* \vec{a} \bullet \vec{b} = \begin{pmatrix} 8 \\ -9 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix} = 8 \cdot 5 + (-9) \cdot (-2) + 1 \cdot 3 = 40 + 18 + 3 = \boxed{61}$

$* \vec{b} \bullet \vec{a} = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix} \bullet \begin{pmatrix} 8 \\ -9 \\ 1 \end{pmatrix} = 5 \cdot 8 + (-2) \cdot (-9) + 3 \cdot 1 = 40 + 18 + 3 = \boxed{61} = \vec{a} \bullet \vec{b}$

$* \vec{a} \bullet \vec{c} = \begin{pmatrix} 8 \\ -9 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 0 \\ -8 \end{pmatrix} = 8 \cdot 1 + (-9) \cdot 0 + 1 \cdot (-8) = 8 + 0 - 8 = \boxed{0} \iff \vec{a} \perp \vec{c}$

$* \vec{c} \bullet \vec{a} = \begin{pmatrix} 1 \\ 0 \\ -8 \end{pmatrix} \bullet \begin{pmatrix} 8 \\ -9 \\ 1 \end{pmatrix} = 1 \cdot 8 + 0 \cdot (-9) + (-8) \cdot 1 = 8 + 0 - 8 = \boxed{0} \iff \vec{c} \perp \vec{a}$

$* \vec{b} \bullet \vec{c} = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 0 \\ -8 \end{pmatrix} = 5 \cdot 1 + (-2) \cdot 0 + 3 \cdot (-8) = 5 + 0 - 24 = \boxed{-19}$

$* \vec{a} \bullet (\vec{b} + \vec{c}) = \begin{pmatrix} 8 \\ -9 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 6 \\ -2 \\ -5 \end{pmatrix} = 8 \cdot 6 + (-9) \cdot (-2) + 1 \cdot (-5) = 48 + 18 - 5 = \boxed{61}$

$* (\vec{c} - \vec{a}) \bullet (\vec{b} - \vec{a}) = \begin{pmatrix} -7 \\ 9 \\ -9 \end{pmatrix} \bullet \begin{pmatrix} -3 \\ 7 \\ 2 \end{pmatrix} = (-7) \cdot (-3) + 9 \cdot 7 + (-9) \cdot 2 = 21 + 63 - 18 = \boxed{66}$

Exercice 21.

a) $* \vec{a} \bullet \vec{d} = \begin{pmatrix}1\\1\\1\end{pmatrix} \bullet \begin{pmatrix}1\\2\\-3\end{pmatrix} = 1\cdot 1 + 1\cdot 2 + 1\cdot(-3) = 1+2-3 = \boxed{0} \iff \vec{a} \perp \vec{d}$

$* \vec{a} \bullet \vec{g} = \begin{pmatrix}1\\1\\1\end{pmatrix} \bullet \begin{pmatrix}1\\1\\-2\end{pmatrix} = 1\cdot 1 + 1\cdot 1 + 1\cdot(-2) = 1+1-2 = \boxed{0} \iff \vec{a} \perp \vec{g}$

$* \vec{b} \bullet \vec{h} = \begin{pmatrix}2\\-1\\4\end{pmatrix} \bullet \begin{pmatrix}-2\\0\\1\end{pmatrix} = 2\cdot(-2) + (-1)\cdot 0 + 4\cdot 1 = -4+0+4 = \boxed{0} \iff \vec{b} \perp \vec{h}$

$* \vec{d} \bullet \vec{i} = \begin{pmatrix}1\\2\\-3\end{pmatrix} \bullet \begin{pmatrix}4\\1\\2\end{pmatrix} = 1\cdot 4 + 2\cdot 1 + (-3)\cdot 2 = 4+2-6 = \boxed{0} \iff \vec{d} \perp \vec{i}$

b) $* (\vec{b} \bullet \vec{c}) \cdot \vec{a} = \left(\begin{pmatrix}2\\-1\\4\end{pmatrix} \bullet \begin{pmatrix}1\\2\\3\end{pmatrix}\right) \cdot \begin{pmatrix}1\\1\\1\end{pmatrix} = 12 \cdot \begin{pmatrix}1\\1\\1\end{pmatrix} = \boxed{\begin{pmatrix}12\\12\\12\end{pmatrix}}$

$* (\vec{b} \bullet \vec{h}) \cdot \vec{c} = \left(\begin{pmatrix}2\\-1\\4\end{pmatrix} \bullet \begin{pmatrix}-2\\0\\1\end{pmatrix}\right) \cdot \begin{pmatrix}1\\2\\3\end{pmatrix} = 0 \cdot \begin{pmatrix}1\\2\\3\end{pmatrix} = \boxed{\begin{pmatrix}0\\0\\0\end{pmatrix}}$

$* (\vec{b} \bullet \vec{c}) \cdot \vec{i} = \left(\begin{pmatrix}2\\-1\\4\end{pmatrix} \bullet \begin{pmatrix}1\\2\\3\end{pmatrix}\right) \cdot \begin{pmatrix}4\\1\\2\end{pmatrix} = 12 \cdot \begin{pmatrix}4\\1\\2\end{pmatrix} = \boxed{\begin{pmatrix}48\\12\\24\end{pmatrix}}$

$* (\vec{g} \bullet \vec{a}) \cdot \vec{d} = \left(\begin{pmatrix}1\\1\\-2\end{pmatrix} \bullet \begin{pmatrix}1\\1\\1\end{pmatrix}\right) \cdot \begin{pmatrix}1\\2\\-3\end{pmatrix} = 0 \cdot \begin{pmatrix}1\\2\\-3\end{pmatrix} = \boxed{\begin{pmatrix}0\\0\\0\end{pmatrix}}$

$* (\vec{a} \bullet \vec{b}) \cdot (\vec{d} \bullet \vec{a}) = \left(\begin{pmatrix}1\\1\\1\end{pmatrix} \bullet \begin{pmatrix}2\\-1\\1\end{pmatrix}\right) \cdot \left(\begin{pmatrix}1\\2\\-3\end{pmatrix} \bullet \begin{pmatrix}1\\1\\1\end{pmatrix}\right) = 2 \cdot 0 = \boxed{0}$

Exercice 22.

$* \overrightarrow{AB} = \begin{pmatrix}6\\4\end{pmatrix}$; $\overrightarrow{BC} = \begin{pmatrix}2\\-3\end{pmatrix}$; $\overrightarrow{CD} = \begin{pmatrix}-6\\-4\end{pmatrix}$; $\overrightarrow{AD} = \begin{pmatrix}2\\-3\end{pmatrix}$

$\Rightarrow \overrightarrow{AB} = \overrightarrow{DC}$; $\overrightarrow{BC} = \overrightarrow{AD}$

$\Rightarrow ||\overrightarrow{AB}|| = ||\overrightarrow{CD}|| = \sqrt{52} = 2\sqrt{13}$ [u] et $||\overrightarrow{BC}|| = ||\overrightarrow{AD}|| = \sqrt{13}$ [u]

$* \overrightarrow{AB} \bullet \overrightarrow{BC} = \begin{pmatrix}6\\4\end{pmatrix} \bullet \begin{pmatrix}2\\-3\end{pmatrix} = 6\cdot 2 + 4\cdot(-3) = 12 - 12 = 0 \Rightarrow \overrightarrow{AB} \perp \overrightarrow{BC}$ etc ...

$\Rightarrow$ $ABCD$ est un quadrilatère avec deux paires de côtés parallèles et isométriques et l'angle en chaque sommet est droit donc c'est un rectangle.

Exercice 24.

$\overrightarrow{b}$ est orthogonal à $\overrightarrow{a} \iff \overrightarrow{b} \bullet \overrightarrow{a} = 0 \iff \begin{pmatrix} 2 \\ \lambda \end{pmatrix} \bullet \begin{pmatrix} 3 \\ -8 \end{pmatrix} = 0 \iff$

$\iff 2 \cdot 3 + \lambda \cdot (-8) = 0 \iff 6 - 8\lambda = 0 \iff 8\lambda = 6 \iff \boxed{\lambda = 3/4}$

Exercice 27.

$*$ On pose $\overrightarrow{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ tel que $||\overrightarrow{v}|| = \sqrt{v_1^2 + v_2^2} = 15 \iff v_1^2 + v_2^2 = 225$

$*$ $\overrightarrow{v}$ est orthogonal à $\overrightarrow{a} \iff \overrightarrow{v} \bullet \overrightarrow{a} = 0 \iff \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ -3 \end{pmatrix} = 0 \iff 4 \cdot v_1 - 3 \cdot v_2 = 0$

$*$ on va résoudre le système suivant par substitution :

$$\begin{cases} v_1^2 + v_2^2 = 225 \\ 4v_1 - 3v_2 = 0 \end{cases} \iff \begin{cases} v_1^2 + v_2^2 = 225 \\ v_2 = \dfrac{4}{3} v_1 \end{cases} \Rightarrow$$

$$\Rightarrow v_1^2 + \frac{16}{9} v_1^2 = 225 \iff 9v_1^2 + 16v_1^2 = 2025 \iff 25v_1^2 - 2025 = 0 \iff$$

$$\iff 25(v_1^2 - 81) = 0 \iff 25(v_1 + 9)(v_1 - 9) = 0 \iff v_1 = -9 \text{ ou } v_1' = 9$$

$$\text{et } v_2 = -12 \text{ ou } v_2' = 12 \Rightarrow \boxed{\overrightarrow{v} = \begin{pmatrix} -9 \\ -12 \end{pmatrix} \text{ ou } \overrightarrow{v'} = \begin{pmatrix} 9 \\ 12 \end{pmatrix}}$$

ou autre méthode.

Exercice 28.

$*$ On pose $\overrightarrow{v} = \begin{pmatrix} 0 \\ v_2 \\ v_3 \end{pmatrix}$ tel que $||\overrightarrow{v}|| = \sqrt{0^2 + v_2^2 + v_3^2} = 1 \iff v_2^2 + v_3^2 = 1$

$*$ $\overrightarrow{v}$ est orthogonal à $\overrightarrow{a} \iff \overrightarrow{v} \bullet \overrightarrow{a} = 0 \iff \begin{pmatrix} 0 \\ v_2 \\ v_3 \end{pmatrix} \bullet \begin{pmatrix} 5 \\ -4 \\ 3 \end{pmatrix} = 0 \iff$

$\iff 0 \cdot 5 + (-4) \cdot v_2 + 3 \cdot v_3 = 0 \iff -4v_2 + 3v_3 = 0$

$*$ on va résoudre le système suivant par substitution :

$$\begin{cases} v_2^2 + v_3^2 = 1 \\ -4v_2 + 3v_3 = 0 \end{cases} \iff \begin{cases} v_2^2 + v_3^2 = 1 \\ v_3 = \dfrac{4}{3}\, v_2 \end{cases} \Rightarrow$$

$$\Rightarrow v_2^2 + \frac{16}{9}\, v_2^2 = 1 \iff 9v_2^2 + 16v_2^2 = 9 \iff 25v_2^2 - 9 = 0 \iff$$

$$\iff (5v_2 + 3)(5v_2 - 3) = 0 \iff v_2 = -\frac{3}{5} \text{ ou } v_2' = \frac{3}{5}$$

$$\text{et } v_3 = -\frac{4}{5} \text{ ou } v_2' = \frac{4}{5} \Rightarrow \boxed{\overrightarrow{v} = \begin{pmatrix} 0 \\ -3/5 \\ -4/5 \end{pmatrix} \text{ ou } \overrightarrow{v'} = \begin{pmatrix} 0 \\ 3/5 \\ 4/5 \end{pmatrix}}$$

ou autre méthode.

Exercice 29.

$*$ $\begin{pmatrix} 7 \\ s \\ t \end{pmatrix} \bullet \begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix} = 0 \iff 7 \cdot 4 + 3 \cdot s + 8 \cdot t = 0 \iff 3s + 8t = -28$

$*$ $\begin{pmatrix} 7 \\ s \\ t \end{pmatrix} \bullet \begin{pmatrix} -5 \\ 20 \\ 9 \end{pmatrix} = 0 \iff 7 \cdot (-5) + 20 \cdot s + 9 \cdot t = 0 \iff 20s + 9t = 35$

$*$ on va résoudre le système suivant par combinaison linéaire :

$$\begin{cases} 3s + 8t = -28 & | \cdot 9 \\ 20s + 9t = 35 & | \cdot (-8) \end{cases} \iff \begin{cases} 27s + 72t = -252 \\ -160s - 72t = -280 \end{cases} \Rightarrow$$

$$\Rightarrow -133s = -532 \Rightarrow \boxed{s = 4 \quad \text{et} \quad t = -5}$$

Exercice 32.

$\ast$ On a $\overrightarrow{AB} = \begin{pmatrix} -4 \\ -4 \\ 3 \end{pmatrix}$

$\ast$ $P \in Ox \Rightarrow P(x\,;0\,;0) \Rightarrow \overrightarrow{AP} = \begin{pmatrix} x+2 \\ -3 \\ 2 \end{pmatrix}$; $\overrightarrow{BP} = \begin{pmatrix} x+6 \\ 1 \\ -1 \end{pmatrix}$

$\ast$ ABC rectangle en $A \iff \overrightarrow{AB} \perp \overrightarrow{AP} \iff \overrightarrow{AB} \bullet \overrightarrow{AP} = 0 \iff$

$$\iff \begin{pmatrix} -4 \\ -4 \\ 3 \end{pmatrix} \bullet \begin{pmatrix} x+2 \\ -3 \\ 2 \end{pmatrix} = 0 \iff (-4)\cdot(x+2) + (-4)\cdot(-3) + 3\cdot 2 = 0 \iff$$

$$\iff -4x - 8 + 12 + 6 = 0 \iff 4x = 10 \iff x_1 = 5/2 \Rightarrow \boxed{P_1(5/2\,;0\,;0)}$$

$\ast$ ABC rectangle en $B \iff \overrightarrow{AB} \perp \overrightarrow{BP} \iff \overrightarrow{AB} \bullet \overrightarrow{BP} = 0 \iff$

$$\iff \begin{pmatrix} -4 \\ -4 \\ 3 \end{pmatrix} \bullet \begin{pmatrix} x+6 \\ 1 \\ -1 \end{pmatrix} = 0 \iff (-4)\cdot(x+6) + (-4)\cdot 1 + 3\cdot(-1) = 0 \iff$$

$$\iff -4x - 24 - 4 - 3 = 0 \iff 4x = -31 \iff x_2 = -31/4 \Rightarrow \boxed{P_2(-31/4\,;0\,;0)}$$

$\ast$ ABC rectangle en $P \iff \overrightarrow{AP} \perp \overrightarrow{BP} \iff \overrightarrow{AP} \bullet \overrightarrow{BP} = 0 \iff$

$$\iff \begin{pmatrix} x+2 \\ -3 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} x+6 \\ 1 \\ -1 \end{pmatrix} = 0 \iff (x+2)\cdot(x+6) + (-3)\cdot 1 + 2\cdot(-1) = 0 \iff$$

$$\iff x^2 + 8x + 12 - 3 - 2 = 0 \iff x^2 + 8x + 7 = 0 \iff (x+7)(x+1) = 0 \iff$$

$$\iff x_3 = -7 \text{ ou } x_4 = -1 \Rightarrow \boxed{P_3(-7\,;0\,;0) \text{ ou } P_4(-1\,;0\,;0)}$$

Exercice 33.

$*$ On a $\overrightarrow{AB} = \begin{pmatrix} 1 \\ -6 \end{pmatrix}$

$*$ Soit $C(x\,;y) \Rightarrow \overrightarrow{AC} = \begin{pmatrix} x-2 \\ y-1 \end{pmatrix}$; $\overrightarrow{BC} = \begin{pmatrix} x-3 \\ y+5 \end{pmatrix}$

a) $ABCD$ est un carré :

1) $\overrightarrow{AB} \perp \overrightarrow{BC} \iff \overrightarrow{AB} \bullet \overrightarrow{BC} = 0 \iff$

$$\iff \begin{pmatrix} 1 \\ -6 \end{pmatrix} \bullet \begin{pmatrix} x-3 \\ y+5 \end{pmatrix} = 0 \iff 1 \cdot (x-3) + (-6) \cdot (y+5) = 0 \iff$$

$$\iff x - 3 - 6y - 30 = 0 \iff x - 6y - 33 = 0$$

2) $||\overrightarrow{AB}|| = ||\overrightarrow{BC}|| \iff$

$$\iff \sqrt{1^2 + (-6)^2} = \sqrt{(x-3)^2 + (y+5)^2} \overset{()^2}{\Rightarrow}$$

$$\overset{()^2}{\Rightarrow} 37 = x^2 - 6x + 9 + y^2 + 10y + 25 \iff$$

$$\iff x^2 - 6x + y^2 + 10y - 3 = 0$$

3) on va résoudre le système suivant par substitution :

$$\begin{cases} x - 6y - 33 = 0 \\ x^2 - 6x + y^2 + 10y - 3 = 0 \end{cases} \iff \begin{cases} x = 6y + 33 \\ (6y+33)^2 - 6(6y+33) + y^2 + 10y - 3 = 0 \end{cases} \Rightarrow$$

$$\Rightarrow 36y^2 + 396y + 1089 - 36y - 198 + y^2 + 10y - 3 = 0 \iff 37y^2 + 370y + 888 = 0 \iff$$

$$\iff 37(y^2 + 10y + 24) = 0 \iff 37(y+6)(y+4)) = 0 \iff y_1 = -6 \text{ ou } y_2 = -4$$

et $x_1 = -3$ ou $x_2 = 9 \Rightarrow \boxed{C_1(-3\,;-6) \text{ ou } C_2(9\,;-4)}$

4) Soit $D_1(u_1\,;v_1)$

$$\overrightarrow{BA} = \overrightarrow{C_1D_1} \iff \begin{pmatrix} -1 \\ 6 \end{pmatrix} = \begin{pmatrix} u_1+3 \\ v_1+6 \end{pmatrix} \Rightarrow u_1 = -4 \text{ et } v_1 = 0 \Rightarrow \boxed{D_1(-4\,;0)}$$

5) Soit $D_2(u_2\,;v_2)$

$$\overrightarrow{BA} = \overrightarrow{C_2D_2} \iff \begin{pmatrix} -1 \\ 6 \end{pmatrix} = \begin{pmatrix} u_2-9 \\ v_2+4 \end{pmatrix} \Rightarrow u_2 = 8 \text{ et } v_2 = 2 \Rightarrow \boxed{D_2(8\,;2)}$$

ou autre méthode ...

b) $ACBD$ est un carré :

1) $\overrightarrow{AC} \perp \overrightarrow{BC} \iff \overrightarrow{AC} \bullet \overrightarrow{BC} = 0 \iff$

$$\iff \begin{pmatrix} x-2 \\ y-1 \end{pmatrix} \bullet \begin{pmatrix} x-3 \\ y+5 \end{pmatrix} = 0 \iff (x-2)\cdot(x-3) + (y-1)\cdot(y+5) = 0 \iff$$

$$\iff x^2 - 5x + 6 + y^2 + 4y - 5 = 0 \iff x^2 - 5x + y^2 + 4y + 1 = 0$$

2) $||\overrightarrow{AC}|| = ||\overrightarrow{BC}|| \iff$

$$\iff \sqrt{(x-2)^2 + (y-1)^2} = \sqrt{(x-3)^2 + (y+5)^2} \overset{()^2}{\Rightarrow}$$

$$\overset{()^2}{\Rightarrow} x^2 - 4x + 4 + y^2 - 2y + 1 = x^2 - 6x + 9 + y^2 + 10y + 25 \iff$$

$$\iff 2x - 12y - 29 = 0$$

3) on va résoudre le système suivant par substitution :

$$\begin{cases} x^2 - 5x + y^2 + 4y + 1 = 0 \\ 2x - 12y - 29 = 0 \end{cases} \iff \begin{cases} \left(\dfrac{12y+29}{2}\right)^2 - 5\left(\dfrac{12y+29}{2}\right) + y^2 + 4y + 1 = 0 \\ x = \dfrac{12y+29}{2} \end{cases}$$

$$\Rightarrow \frac{144y^2 + 696y + 841}{4} - \frac{60y + 145}{2} + y^2 + 4y + 1 = 0 \overset{\cdot 4}{\iff}$$

$$\iff 144y^2 + 696y + 841 - 120y - 290 + 4y^2 + 16y + 4 = 0 \iff 148y^2 + 592y + 555 = 0 \iff$$

$$\iff 37(4y^2 + 16y + 15) = 0 \iff 37(2y+5)(2y+3)) = 0 \iff y_1 = -5/2 \text{ ou } y_2 = -3/2$$

et $x_1 = -1/2$ ou $x_2 = 11/2 \Rightarrow$ $\boxed{C(-1/2\,;-5/2) \text{ ou } D(11/2\,;-3/2)}$

ou autre méthode ...

Exercice 34.

$*$ On a $\overrightarrow{BC} = \begin{pmatrix} -2 \\ -6 \end{pmatrix}$

$*$ Soit $A(x\,;y) \Rightarrow \overrightarrow{BA} = \begin{pmatrix} x-3 \\ y-4 \end{pmatrix} \quad ; \quad \overrightarrow{CA} = \begin{pmatrix} x-1 \\ y+2 \end{pmatrix}$

ABC est un triangle rectangle et isocèle en A :

1) $\overrightarrow{BA} \perp \overrightarrow{CA} \iff \overrightarrow{BA} \bullet \overrightarrow{CA} = 0 \iff$

$$\iff \begin{pmatrix} x-3 \\ y-4 \end{pmatrix} \bullet \begin{pmatrix} x-1 \\ y+2 \end{pmatrix} = 0 \iff (x-3)\cdot(x-1) + (y-4)\cdot(y+2) = 0 \iff$$

$$\iff x^2 - 4x + 3 + y^2 - 2y - 8 = 0 \iff x^2 - 4x + y^2 - 2y - 5 = 0$$

2) $||\overrightarrow{BA}|| = ||\overrightarrow{CA}|| \iff$

$$\iff \sqrt{(x-3)^2 + (y-4)^2} = \sqrt{(x-1)^2 + (y+2)^2} \overset{()^2}{\Rightarrow}$$

$$\overset{()^2}{\Rightarrow} x^2 - 6x + 9 + y^2 - 8y + 16 = x^2 - 2x + 1 + y^2 + 4y + 4 \iff$$

$$\iff 4x + 12y - 20 = 0 \iff x + 3y - 5 = 0$$

3) on va résoudre le système suivant par substitution :

$$\begin{cases} x^2 - 4x + y^2 - 2y - 5 = 0 \\ x + 3y - 5 = 0 \end{cases} \iff \begin{cases} (-3y+5)^2 - 4(-3y+5) + y^2 - 2y - 5 = 0 \\ x = -3y + 5 \end{cases} \Rightarrow$$

$$\Rightarrow 9y^2 - 30y + 25 + 12y - 20 + y^2 - 2y - 5 = 0 \iff 10y^2 - 20y = 0 \iff$$

$$\iff 10y(y-2) = 0 \iff y_1 = 0 \text{ ou } y_2 = 2$$

$$\text{et } x_1 = 5 \text{ ou } x_2 = -1 \Rightarrow \boxed{A_1(5\,;0) \text{ ou } A_2(-1\,;2)}$$

ou autre méthode ...

Exercice 35.

$*$ On a $\overrightarrow{AB} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$

$*$ $C(\lambda\,;\lambda) \Rightarrow \overrightarrow{AC} = \begin{pmatrix} \lambda+2 \\ \lambda-4 \end{pmatrix}$; $\overrightarrow{BC} = \begin{pmatrix} \lambda-1 \\ \lambda+2 \end{pmatrix}$

1) ABC est un triangle rectangle en A :

$$\overrightarrow{AB} \perp \overrightarrow{AC} \iff \overrightarrow{AB} \bullet \overrightarrow{AC} = 0 \iff \begin{pmatrix} 3 \\ -6 \end{pmatrix} \bullet \begin{pmatrix} \lambda+2 \\ \lambda-4 \end{pmatrix} = 0 \iff$$

$$\iff 3 \cdot (\lambda+2) + (-6) \cdot (\lambda-4) = 0 \iff 3\lambda + 6 - 6\lambda + 24 = 0 \iff$$

$$\iff -3\lambda + 30 = 0 \iff 3\lambda = 30 \iff \boxed{\lambda_1 = 10}$$

Remarque : $||\overrightarrow{AB}|| = \sqrt{45} = 3\sqrt{5}$; $||\overrightarrow{AC}|| = ||\begin{pmatrix} 12 \\ 6 \end{pmatrix}|| = \sqrt{180} = 6\sqrt{5}$

2) ABC est un triangle rectangle en B :

$$\overrightarrow{AB} \perp \overrightarrow{BC} \iff \overrightarrow{AB} \bullet \overrightarrow{BC} = 0 \iff \begin{pmatrix} 3 \\ -6 \end{pmatrix} \bullet \begin{pmatrix} \lambda-1 \\ \lambda+2 \end{pmatrix} = 0 \iff$$

$$\iff 3 \cdot (\lambda-1) + (-6) \cdot (\lambda+2) = 0 \iff 3\lambda - 3 - 6\lambda - 12 = 0 \iff$$

$$\iff -3\lambda - 15 = 0 \iff 3\lambda = -15 \iff \boxed{\lambda_2 = -5}$$

Remarque : $||\overrightarrow{AB}|| = \sqrt{45} = 3\sqrt{5}$; $||\overrightarrow{BC}|| = ||\begin{pmatrix} -6 \\ -3 \end{pmatrix}|| = \sqrt{45} = 3\sqrt{5}$

Pour $\lambda_2 = -5$, ABC sera rectangle et isocèle en B.

3) ABC est un triangle rectangle en C :

$$\overrightarrow{AC} \perp \overrightarrow{BC} \iff \overrightarrow{AC} \bullet \overrightarrow{BC} = 0 \iff \begin{pmatrix} \lambda+2 \\ \lambda-4 \end{pmatrix} \bullet \begin{pmatrix} \lambda-1 \\ \lambda+2 \end{pmatrix} = 0 \iff$$

$$\iff (\lambda+2) \cdot (\lambda-1) + (\lambda-4) \cdot (\lambda+2) = 0 \iff \lambda^2 + \lambda - 2 + \lambda^2 - 2\lambda - 8 = 0 \iff$$

$$\iff 2\lambda^2 - \lambda - 10 = 0 \iff (2\lambda-5)(\lambda+2) = 0 \iff \boxed{\lambda_3 = 5/2 \text{ ou } \lambda_4 = -2}$$

Remarques :

- $\lambda_3 = 5/2$: $||\overrightarrow{AC}|| = ||\begin{pmatrix} 9/2 \\ -3/2 \end{pmatrix}|| = \sqrt{90/4} = 3\sqrt{10}/2$

$||\overrightarrow{BC}|| = ||\begin{pmatrix} 3/2 \\ 9/2 \end{pmatrix}|| = 3\sqrt{10}/2 \Rightarrow ABC$ sera rectangle et isocèle en C.

- $\lambda_4 = -2$: $||\overrightarrow{AC}|| = ||\begin{pmatrix} 0 \\ -6 \end{pmatrix}|| = \sqrt{36} = 6$; $||\overrightarrow{BC}|| = ||\begin{pmatrix} -3 \\ 0 \end{pmatrix}|| = \sqrt{9} = 3$

Exercice 39.

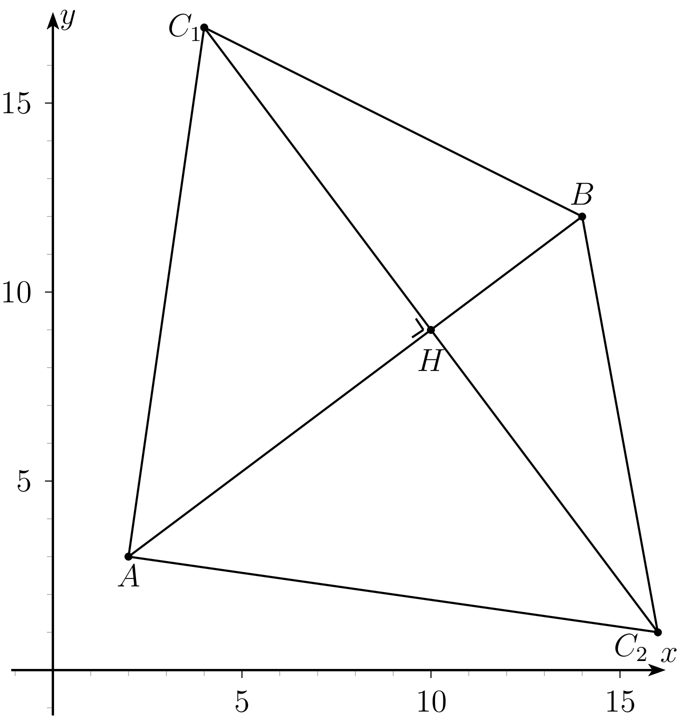
• $\overrightarrow{AB} = \begin{pmatrix} 12 \\ 9 \end{pmatrix}$; $\overrightarrow{AH} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$

a) A, B et H sont alignés $\overset{\text{ex.59 p.40}}{\Longleftrightarrow}$ $\overrightarrow{AB}$ et $\overrightarrow{AH}$ sont colinéaires $\overset{\text{p.22}}{\Longleftrightarrow}$ $\det(\overrightarrow{AB}\,;\overrightarrow{AH}) = 0$

$\det(\overrightarrow{AB}\,;\overrightarrow{AH}) = \begin{vmatrix} 12 & 8 \\ 9 & 6 \end{vmatrix} = 12 \cdot 6 - 9 \cdot 8 = 72 - 72 = 0 \iff$

$\iff$ $\overrightarrow{AB}$ et $\overrightarrow{AH}$ sont colinéaires $\iff$ A, B et H sont alignés

b) Figure d'étude :



• $||\overrightarrow{AB}|| = \sqrt{225} = 15\,[\text{u}]$

• $\sigma(\Delta ABC) = \dfrac{||\overrightarrow{AB}|| \cdot ||\overrightarrow{CH}||}{2} = \dfrac{1}{2} \cdot 15 \cdot ||\overrightarrow{CH}|| = 75\,[\text{u}^2] \Rightarrow ||\overrightarrow{CH}|| = 10\,[\text{u}]$

• On pose $C(c_1; c_2)$

• $\vec{v} = \overrightarrow{AB}_u = \dfrac{\overrightarrow{AB}}{||\overrightarrow{AB}||} = \dfrac{1}{15}\begin{pmatrix} 12 \\ 9 \end{pmatrix} = \begin{pmatrix} 4/5 \\ 3/5 \end{pmatrix} \Rightarrow v_\perp = \begin{pmatrix} -3/5 \\ 4/5 \end{pmatrix}$

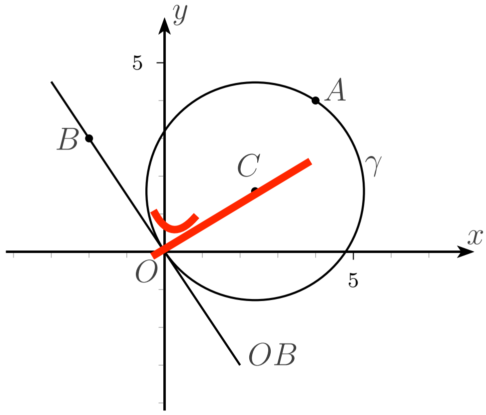
• $\overrightarrow{HC} = \pm 10 \cdot \vec{v_\perp} \Rightarrow \begin{pmatrix} c_1 - 10 \\ c_2 - 9 \end{pmatrix} = \pm 10 \cdot \begin{pmatrix} -3/5 \\ 4/5 \end{pmatrix} = \pm \begin{pmatrix} -6 \\ 8 \end{pmatrix}$

$+ \Rightarrow$ $\boxed{C_1\,(4\,;17)}$

$- \Rightarrow$ $\boxed{C_2\,(16\,;1)}$

Exercice 40.

Figure d'étude :



$*$ On pose $C(x;y)$

$* \overrightarrow{AC} = \begin{pmatrix} x-4 \\ y-4 \end{pmatrix}$; $\overrightarrow{OC} = \begin{pmatrix} x-0 \\ y-0 \end{pmatrix}$

$* r = ||\overrightarrow{AC}|| = ||\overrightarrow{OC}|| \iff$

$\iff \sqrt{(x-4)^2 + (y-4)^2} = \sqrt{x^2+y^2} \overset{()^2}{\Rightarrow}$

$\overset{()^2}{\Rightarrow} (x-4)^2 + (y-4)^2 = x^2 + y^2 \iff$

$\iff x^2 - 8x + 16 + y^2 - 8y + 16 = x^2 + y^2 \iff$

$\iff 8x + 8y - 32 = 0 \iff x + y - 4 = 0$

$* \overrightarrow{OB} \perp \overrightarrow{OC} \iff \overrightarrow{OB} \bullet \overrightarrow{OC} = 0 \iff$

$\iff \begin{pmatrix} -2 \\ 3 \end{pmatrix} \bullet \begin{pmatrix} x \\ y \end{pmatrix} = 0 \iff -2x + 3y = 0$

$*$ on va résoudre le système suivant par substitution :

$$\begin{cases} x + y - 4 = 0 \\ -2x + 3y = 0 \end{cases} \iff \begin{cases} x = -y + 4 \\ -2(-y+4) + 3y = 0 \end{cases} \Rightarrow$$

$$\Rightarrow 2y - 8 + 3y = 0 \iff 5y = 8 \iff y = 8/5$$

$$\text{et } x = 12/5 \Rightarrow \boxed{C(12/5\,;8/5)}$$

$* r = ||\overrightarrow{AC}|| = \sqrt{(-8/5)^2 + (-12/5)^2} = \sqrt{208/25} = \dfrac{4\sqrt{13}}{5}$ [u]

ou autre méthode ...

Exercice 51.

a) $* \overrightarrow{a} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad ; \quad \overrightarrow{b} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$

$* \; ||\overrightarrow{a}|| = \sqrt{10}\,[u] \quad ; \quad ||\overrightarrow{b}|| = \sqrt{45} = 3\sqrt{5}\,[u]$

$* \; \overrightarrow{a} \bullet \overrightarrow{b} = 18 - 3 = 15$

$$* \; \cos(\alpha) = \frac{\overrightarrow{a} \bullet \overrightarrow{b}}{||\overrightarrow{a}|| \cdot ||\overrightarrow{b}||} = \frac{15}{\sqrt{10 \cdot 45}} = \frac{15}{15\sqrt{2}} \Rightarrow \alpha = \arccos\left(\frac{1}{\sqrt{2}}\right) = \boxed{45^\circ}$$

b) $* \overrightarrow{a} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} \quad ; \quad \overrightarrow{b} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

$* \; ||\overrightarrow{a}|| = \sqrt{25} = 5\,[u] \quad ; \quad ||\overrightarrow{b}|| = \sqrt{26}\,[u]$

$* \; \overrightarrow{a} \bullet \overrightarrow{b} = -15 - 4 = -19$

$$* \; \cos(\alpha) = \frac{\overrightarrow{a} \bullet \overrightarrow{b}}{||\overrightarrow{a}|| \cdot ||\overrightarrow{b}||} = \frac{-19}{5\sqrt{26}} \Rightarrow \alpha = \arccos\left(\frac{-19}{5\sqrt{26}}\right) \cong \boxed{138.18^\circ}$$

c) $* \overrightarrow{a} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} \quad ; \quad \overrightarrow{b} = \begin{pmatrix} 6 \\ 15 \end{pmatrix}$

$* \; ||\overrightarrow{a}|| = \sqrt{29}\,[u] \quad ; \quad ||\overrightarrow{b}|| = \sqrt{261}\,[u]$

$* \; \overrightarrow{a} \bullet \overrightarrow{b} = 30 - 30 = 0 \Rightarrow \overrightarrow{a} \perp \overrightarrow{b} \Rightarrow \alpha = \boxed{90^\circ}$

Exercice 52.

a) $* \ \overrightarrow{a} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \quad ; \quad \overrightarrow{b} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$

$* \ ||\overrightarrow{a}|| = \sqrt{9} = 3\,[u] \quad ; \quad ||\overrightarrow{b}|| = \sqrt{9} = 3\,[u]$

$* \ \overrightarrow{a} \bullet \overrightarrow{b} = 2 - 2 + 4 = 4$

$$* \ \cos(\alpha) = \frac{\overrightarrow{a} \bullet \overrightarrow{b}}{||\overrightarrow{a}|| \cdot ||\overrightarrow{b}||} = \frac{4}{3 \cdot 3} = \frac{4}{9} \Rightarrow \alpha = \arccos\left(\frac{4}{9}\right) \cong \boxed{63.61^\circ}$$

b) $* \ \overrightarrow{a} = \begin{pmatrix} 3 \\ -4 \\ -1 \end{pmatrix} \quad ; \quad \overrightarrow{b} = \begin{pmatrix} -2 \\ 3 \\ -2 \end{pmatrix}$

$* \ ||\overrightarrow{a}|| = \sqrt{26}\,[u] \quad ; \quad ||\overrightarrow{b}|| = \sqrt{17}\,[u]$

$* \ \overrightarrow{a} \bullet \overrightarrow{b} = -6 - 12 + 2 = -16$

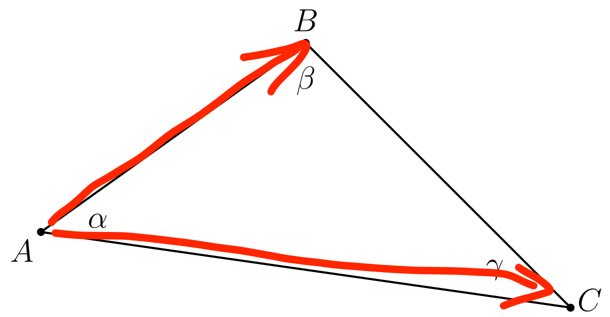
$$* \ \cos(\alpha) = \frac{\overrightarrow{a} \bullet \overrightarrow{b}}{||\overrightarrow{a}|| \cdot ||\overrightarrow{b}||} = \frac{-16}{\sqrt{26 \cdot 17}} \Rightarrow \alpha = \arccos\left(\frac{-16}{\sqrt{442}}\right) \cong \boxed{139.56^\circ}$$

c) $* \ \overrightarrow{a} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \quad ; \quad \overrightarrow{b} = \begin{pmatrix} 6 \\ 6 \\ -2 \end{pmatrix}$

$* \ ||\overrightarrow{a}|| = \sqrt{14}\,[u] \quad ; \quad ||\overrightarrow{b}|| = \sqrt{76} = 2\sqrt{19}\,[u]$

$* \ \overrightarrow{a} \bullet \overrightarrow{b} = 12 - 6 - 6 = 0 \Rightarrow \overrightarrow{a} \perp \overrightarrow{b} \Rightarrow \alpha = \boxed{90^\circ}$

Exercice 55.



$* \ \overrightarrow{AB} = \begin{pmatrix} -4 \\ 10 \\ -6 \end{pmatrix} \quad ; \quad \overrightarrow{AC} = \begin{pmatrix} -8 \\ 5 \\ -3 \end{pmatrix} \quad ; \quad \overrightarrow{BC} = \begin{pmatrix} -4 \\ -5 \\ 3 \end{pmatrix}$

$* \ ||\overrightarrow{AB}|| = \sqrt{152} = 2\sqrt{38}\,[u] \quad ; \quad ||\overrightarrow{AC}|| = \sqrt{98} = 7\sqrt{2}\,[u] \quad ; \quad ||\overrightarrow{BC}|| = \sqrt{50} = 5\sqrt{2}\,[u]$

$* \ \overrightarrow{AB} \bullet \overrightarrow{AC} = 32 + 50 + 18 = 100$

$* \ \cos(\alpha) = \dfrac{\overrightarrow{AB} \bullet \overrightarrow{AC}}{||\overrightarrow{AB}|| \cdot ||\overrightarrow{AC}||} = \dfrac{100}{14\sqrt{38 \cdot 2}} = \dfrac{100}{28\sqrt{19}} = \dfrac{25}{7\sqrt{19}} \Rightarrow$

$\Rightarrow \alpha = \arccos\left(\dfrac{25}{7\sqrt{19}}\right) \cong \boxed{34.98^\circ}$

$* \ \overrightarrow{BA} \bullet \overrightarrow{BC} = -16 + 50 + 18 = 52$

$* \ \cos(\beta) = \dfrac{\overrightarrow{BA} \bullet \overrightarrow{BC}}{||\overrightarrow{AB}|| \cdot ||\overrightarrow{BC}||} = \dfrac{52}{10\sqrt{38 \cdot 2}} = \dfrac{52}{20\sqrt{19}} = \dfrac{13}{5\sqrt{19}} \Rightarrow$

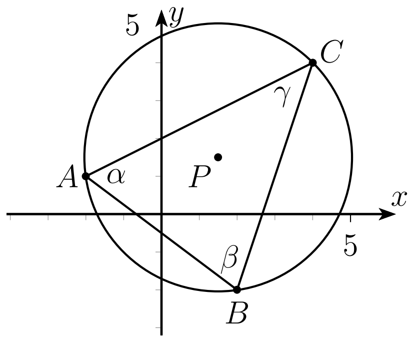
$\Rightarrow \beta = \arccos\left(\dfrac{13}{5\sqrt{19}}\right) \cong \boxed{53.38^\circ}$

$* \ \overrightarrow{CA} \bullet \overrightarrow{CB} = 32 - 25 - 9 = -2$

$* \ \cos(\gamma) = \dfrac{\overrightarrow{CA} \bullet \overrightarrow{CB}}{||\overrightarrow{AC}|| \cdot ||\overrightarrow{BC}||} = \dfrac{-2}{35\sqrt{2 \cdot 2}} = \dfrac{-2}{70} = -\dfrac{1}{35} \Rightarrow$

$\Rightarrow \gamma = \arccos\left(-\dfrac{1}{35}\right) \cong \boxed{91.64^\circ}$

Exercice 56.



a) $* \ \overrightarrow{AB} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \quad ; \quad \overrightarrow{AC} = \begin{pmatrix} 6 \\ 3 \end{pmatrix} \quad ; \quad \overrightarrow{BC} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$

$* \ ||\overrightarrow{AB}|| = \sqrt{25} = 5 \, [u] \quad ; \quad ||\overrightarrow{AC}|| = \sqrt{45} = 3\sqrt{5} \, [u] \quad ; \quad ||\overrightarrow{BC}|| = \sqrt{40} = 2\sqrt{10} \, [u]$

b) $* \ \overrightarrow{AP} = \begin{pmatrix} 7/2 \\ 1/2 \end{pmatrix} \Rightarrow ||\overrightarrow{AP}|| = \sqrt{50/4} = 2.5\sqrt{2} \, [\text{u}]$

$* \ \overrightarrow{BP} = \begin{pmatrix} -1/2 \\ 7/2 \end{pmatrix} \Rightarrow ||\overrightarrow{BP}|| = \sqrt{50/4} = 2.5\sqrt{2} \, [\text{u}]$

$* \ \overrightarrow{CP} = \begin{pmatrix} -5/2 \\ -5/2 \end{pmatrix} \Rightarrow ||\overrightarrow{CP}|| = \sqrt{50/4} = 2.5\sqrt{2} \, [\text{u}]$

$\Rightarrow ||\overrightarrow{AP}|| = ||\overrightarrow{BP}|| = ||\overrightarrow{CP}|| = 2.5\sqrt{2} \, [\text{u}]$

P est donc équidistant à A, B et C donc P est le centre du cercle circonscrit au triangle ABC.

c) $\sigma(\Delta ABC) = \frac{1}{2} \cdot |\det(\overrightarrow{AB} ; \overrightarrow{AC})| = \frac{1}{2} \cdot \begin{vmatrix} 4 & 6 \\ -3 & 3 \end{vmatrix} = \frac{1}{2} \cdot [4 \cdot 3 - (-3) \cdot 6] = \frac{1}{2} \cdot 30 = 15 \, [u^2]$

d) $* \ \overrightarrow{AB} \bullet \overrightarrow{AC} = 24 - 9 = 15$

$* \ \cos(\alpha) = \dfrac{\overrightarrow{AB} \bullet \overrightarrow{AC}}{||\overrightarrow{AB}|| \cdot ||\overrightarrow{AC}||} = \dfrac{15}{5 \cdot 3\sqrt{5}} = \dfrac{1}{\sqrt{5}} \Rightarrow \alpha = \arccos\left(\dfrac{1}{\sqrt{5}}\right) \cong \boxed{63.43^\circ}$

$* \ \overrightarrow{BA} \bullet \overrightarrow{BC} = -8 + 18 = 10$

$* \ \cos(\beta) = \dfrac{\overrightarrow{BA} \bullet \overrightarrow{BC}}{||\overrightarrow{AB}|| \cdot ||\overrightarrow{BC}||} = \dfrac{10}{5 \cdot 2\sqrt{10}} = \dfrac{1}{\sqrt{10}} \Rightarrow \beta = \arccos\left(\dfrac{1}{\sqrt{10}}\right) \cong \boxed{71.57^\circ}$

$* \ \overrightarrow{CA} \bullet \overrightarrow{CB} = 12 + 18 = 30$

$$* \ \cos(\gamma) = \frac{\overrightarrow{CA} \bullet \overrightarrow{CB}}{||\overrightarrow{AC}|| \cdot ||\overrightarrow{BC}||} = \frac{30}{6\sqrt{5 \cdot 10}} = \frac{30}{30\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow$$

$$\Rightarrow \gamma = \arccos\left(\frac{1}{\sqrt{2}}\right) = \boxed{45^\circ}$$