

Exercice 21.

a) • $f(x) = 3x^3 - x^2 + x + 1$; $g(x) = 2x^2 - 1$

• division euclidienne :
début :

$$\begin{array}{r|l} 3x^3 & -x^2 + x + 1 \\ - (3x^3 & + 0x^2 - \frac{3}{2}x) \\ \hline & \frac{5}{2}x + 1 \end{array} \quad \begin{array}{l} 2x^2 + 0x - 1 \\ \hline \frac{3}{2}x - \dots \end{array}$$

etc...

en entier :

$$\begin{array}{r|l} 3x^3 - x^2 + x + 1 & 2x^2 - 1 \\ - 3x^3 & + \frac{3}{2}x \\ \hline & -x^2 + \frac{5}{2}x + 1 \\ & x^2 & - \frac{1}{2} \\ \hline & \frac{5}{2}x + \frac{1}{2} \end{array}$$

• Egalité fondamentale :
$$f(x) = (2x^2 - 1) \left(\frac{3}{2}x - \frac{1}{2} \right) + \frac{5}{2}x + \frac{1}{2}$$

b) • $f(x) = 2x^3 - 1$; $g(x) = 3x^3 - x^2 + x + 1$

• division euclidienne :

$$\begin{array}{r|l} 2x^3 & -1 \\ - 2x^3 + \frac{2}{3}x^2 - \frac{2}{3}x - \frac{2}{3} & \frac{2}{3} \\ \hline & \frac{2}{3}x^2 - \frac{2}{3}x - \frac{5}{3} \end{array}$$

• Egalité fondamentale :
$$f(x) = \frac{2}{3}(3x^3 - x^2 + x + 1) + \frac{2}{3}x^2 - \frac{2}{3}x - \frac{5}{3}$$

c) • $f(x) = 7x^5 - x^4 + 6x^3 - 7x$; $g(x) = 7x^3 - x$

• division euclidienne :

$$\begin{array}{r|l}
 7x^5 - x^4 + 6x^3 & -7x \quad \left| \begin{array}{l} 7x^3 - x \\ x^2 - \frac{1}{7}x + 1 \end{array} \right. \\
 \underline{-7x^5} & +x^3 \\
 -x^4 + 7x^3 & \\
 \underline{x^4} & -\frac{1}{7}x^2 \\
 7x^3 - \frac{1}{7}x^2 - 7x & \\
 \underline{-7x^3} & +x \\
 -\frac{1}{7}x^2 - 6x &
 \end{array}$$

• Egalité fondamentale :
$$f(x) = (7x^3 - x) \left(x^2 - \frac{1}{7}x + 1 \right) - \frac{1}{7}x^2 - 6x$$

d) • $f(x) = 6x^4 + 4x^3 - 7x^2$; $g(x) = 2x^2 - 3$

• division euclidienne :

$$\begin{array}{r|l}
 6x^4 + 4x^3 - 7x^2 & 2x^2 - 3 \\
 \underline{-6x^4} & +9x^2 \\
 4x^3 + 2x^2 & \\
 \underline{-4x^3} & +6x \\
 2x^2 + 6x & \\
 \underline{-2x^2} & +3 \\
 6x + 3 &
 \end{array}$$

• Egalité fondamentale :
$$f(x) = (2x^2 - 3)(3x^2 + 2x + 1) + 6x + 3$$

e) • $f(x) = 14x^4 - 27x^3 + 21x^2 - 3x - 2$; $g(x) = 2x^2 - 3x + 2$

• division euclidienne :

$$\begin{array}{r|l}
 14x^4 - 27x^3 + 21x^2 - 3x - 2 & 2x^2 - 3x + 2 \\
 \underline{-14x^4 + 21x^3 - 14x^2} & \left| \begin{array}{l} 2x^2 - 3x + 2 \\ 7x^2 - 3x - 1 \end{array} \right. \\
 -6x^3 + 7x^2 - 3x & \\
 \underline{6x^3 - 9x^2 + 6x} & \\
 -2x^2 + 3x - 2 & \\
 \underline{2x^2 - 3x + 2} & \\
 0 &
 \end{array}$$

• Egalité fondamentale :
$$f(x) = (2x^2 - 3x + 2)(7x^2 - 3x - 1)$$