

Exercice 29.

- $p(x) = 5x^{100} + 7x^5 - x + 47$; $d(x) = x - 1$
 - $Z_d = \{1\}$
 - $r = p(1) = 5 \cdot 1 + 7 \cdot 1 - 1 + 47 = 58 \Rightarrow \boxed{\text{reste} = 58}$
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Exercice 30.

- $p(x) = 3x^{100} + 5x^{85} - 4x^{38} + 2x^{17} - 6$; $d(x) = x + 1$
 - $Z_d = \{-1\}$
 - $r = p(-1) = 3 \cdot 1 + 5 \cdot (-1) - 4 \cdot 1 + 2 \cdot (-1) - 6 = -14 \Rightarrow \boxed{\text{reste} = -14}$
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Exercice 14.

- $p(x) = x^4 - ax^3 + 3x^2 - 2ax - a^2$ est divisible par $d(x) = x - 2 \iff r = 0$
- $Z_d = \{2\}$
- $r = p(2) = 16 - a \cdot 8 + 3 \cdot 4 - 2a \cdot 2 - a^2 = 0 \iff$
 $\iff a^2 + 12a - 28 = 0 \iff (a + 14)(a - 2) = 0 \Rightarrow \boxed{a_1 = -14 \text{ ou } a_2 = 2}$
- schéma de Horner pour $a_1 = -14$:

$$\begin{array}{r|rrrr}
 & 1 & 14 & 3 & 28 \\
 2 & & 2 & 32 & 70 \\
 \hline
 & 1 & 16 & 35 & 98 \\
 & & & & 0
 \end{array}$$

$$\boxed{q_1(x) = x^3 + 16x^2 + 35x + 98} \quad ; \quad r = 0$$

- schéma de Horner pour $a_2 = 2$:

$$\begin{array}{r|rrrr}
 & 1 & -2 & 3 & -4 \\
 2 & & 2 & 0 & 6 \\
 \hline
 & 1 & 0 & 3 & 2 \\
 & & & & 0
 \end{array}$$

$$\boxed{q_2(x) = x^3 + 3x + 2} \quad ; \quad r = 0$$

Exercice 15.

• $p(x) = ax^3 + x^2 + a^2x + 3a^2 + 11$ est divisible par $d(x) = x + 2 \iff r = 0$

• $Z_d = \{-2\}$

• $r = p(-2) = a \cdot (-8) + 4 + a^2 \cdot (-2) + 3a^2 + 11 = 0 \iff$

$$\iff a^2 - 8a + 15 = 0 \iff (a - 3)(a - 5) = 0 \Rightarrow \boxed{a_1 = 3 \text{ ou } a_2 = 5}$$

• schéma de Horner pour $a_1 = 3$:

$$\begin{array}{r|rrrr} & 3 & 1 & 9 & 38 \\ -2 & & -6 & 10 & -38 \\ \hline & 3 & -5 & 19 & 0 \end{array}$$

$$\boxed{q_1(x) = 3x^2 - 5x + 19} \quad ; \quad r = 0$$

• schéma de Horner pour $a_2 = 5$:

$$\begin{array}{r|rrrr} & 5 & 1 & 25 & 86 \\ -2 & & -10 & 18 & -86 \\ \hline & 5 & -9 & 43 & 0 \end{array}$$

$$\boxed{q_2(x) = 5x^2 - 9x + 43} \quad ; \quad r = 0$$

Exercice 16.

• $p(x) = a^2x^3 - 4ax + 3$ est divisible par $d(x) = x - 1 \iff r = 0$

• $Z_d = \{1\}$

• $r = p(1) = a^2 \cdot 1 - 4a \cdot 1 + 3 = 0 \iff$

$$\iff a^2 - 4a + 3 = 0 \iff (a - 1)(a - 3) = 0 \Rightarrow \boxed{a_1 = 1 \text{ ou } a_2 = 3}$$

• schéma de Horner pour $a_1 = 1$:

$$\begin{array}{r|rrrr} & 1 & 0 & -4 & 3 \\ 1 & & 1 & 1 & -3 \\ \hline & 1 & 1 & -3 & 0 \end{array}$$

$$\boxed{q_1(x) = x^2 + x - 3} \quad ; \quad r = 0$$

• schéma de Horner pour $a_2 = 3$:

$$\begin{array}{r|rrrr} & 9 & 0 & -12 & 3 \\ 1 & & 9 & 9 & -3 \\ \hline & 9 & 9 & -3 & 0 \end{array}$$

$$\boxed{q_2(x) = 9x^2 + 9x - 3} \quad ; \quad r = 0$$