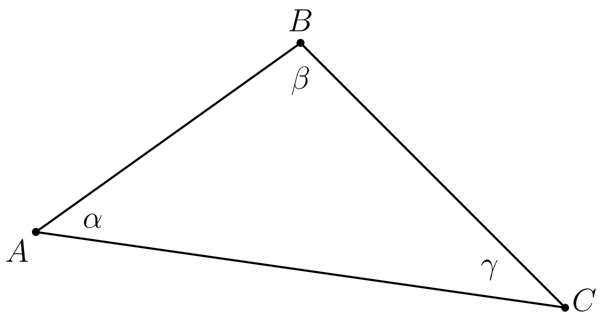
Exercice 55.



$* \ \overrightarrow{AB} = \begin{pmatrix} -4 \\ 10 \\ -6 \end{pmatrix} \quad ; \quad \overrightarrow{AC} = \begin{pmatrix} -8 \\ 5 \\ -3 \end{pmatrix} \quad ; \quad \overrightarrow{BC} = \begin{pmatrix} -4 \\ -5 \\ 3 \end{pmatrix}$

$* \ ||\overrightarrow{AB}|| = \sqrt{152} = 2\sqrt{38}\,[u] \quad ; \quad ||\overrightarrow{AC}|| = \sqrt{98} = 7\sqrt{2}\,[u] \quad ; \quad ||\overrightarrow{BC}|| = \sqrt{50} = 5\sqrt{2}\,[u]$

$* \ \overrightarrow{AB} \bullet \overrightarrow{AC} = 32 + 50 + 18 = 100$

$$* \ \cos(\alpha) = \frac{\overrightarrow{AB} \bullet \overrightarrow{AC}}{||\overrightarrow{AB}|| \cdot ||\overrightarrow{AC}||} = \frac{100}{14\sqrt{38 \cdot 2}} = \frac{100}{28\sqrt{19}} = \frac{25}{7\sqrt{19}} \Rightarrow$$

$$\Rightarrow \alpha = \arccos\left(\frac{25}{7\sqrt{19}}\right) \cong \boxed{34.98^\circ}$$

$* \ \overrightarrow{BA} \bullet \overrightarrow{BC} = -16 + 50 + 18 = 52$

$$* \ \cos(\beta) = \frac{\overrightarrow{BA} \bullet \overrightarrow{BC}}{||\overrightarrow{AB}|| \cdot ||\overrightarrow{BC}||} = \frac{52}{10\sqrt{38 \cdot 2}} = \frac{52}{20\sqrt{19}} = \frac{13}{5\sqrt{19}} \Rightarrow$$

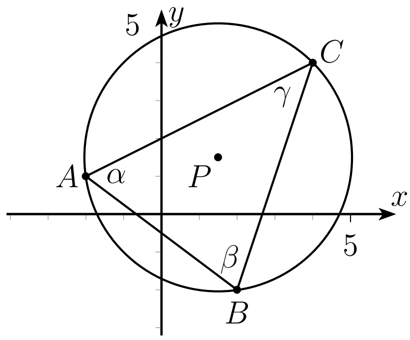
$$\Rightarrow \beta = \arccos\left(\frac{13}{5\sqrt{19}}\right) \cong \boxed{53.38^\circ}$$

$* \ \overrightarrow{CA} \bullet \overrightarrow{CB} = 32 - 25 - 9 = -2$

$$* \ \cos(\gamma) = \frac{\overrightarrow{CA} \bullet \overrightarrow{CB}}{||\overrightarrow{AC}|| \cdot ||\overrightarrow{BC}||} = \frac{-2}{35\sqrt{2 \cdot 2}} = \frac{-2}{70} = -\frac{1}{35} \Rightarrow$$

$$\Rightarrow \gamma = \arccos\left(-\frac{1}{35}\right) \cong \boxed{91.64^\circ}$$

Exercice 56.



a) $* \ \overrightarrow{AB} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \quad ; \quad \overrightarrow{AC} = \begin{pmatrix} 6 \\ 3 \end{pmatrix} \quad ; \quad \overrightarrow{BC} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$

$* \ ||\overrightarrow{AB}|| = \sqrt{25} = 5\,[u] \quad ; \quad ||\overrightarrow{AC}|| = \sqrt{45} = 3\sqrt{5}\,[u] \quad ; \quad ||\overrightarrow{BC}|| = \sqrt{40} = 2\sqrt{10}\,[u]$

b) $* \ \overrightarrow{AP} = \begin{pmatrix} 7/2 \\ 1/2 \end{pmatrix} \Rightarrow ||\overrightarrow{AP}|| = \sqrt{50/4} = 2.5\sqrt{2}\,[\text{u}]$

$* \ \overrightarrow{BP} = \begin{pmatrix} -1/2 \\ 7/2 \end{pmatrix} \Rightarrow ||\overrightarrow{BP}|| = \sqrt{50/4} = 2.5\sqrt{2}\,[\text{u}]$

$* \ \overrightarrow{CP} = \begin{pmatrix} -5/2 \\ -5/2 \end{pmatrix} \Rightarrow ||\overrightarrow{CP}|| = \sqrt{50/4} = 2.5\sqrt{2}\,[\text{u}]$

$\Rightarrow ||\overrightarrow{AP}|| = ||\overrightarrow{BP}|| = ||\overrightarrow{CP}|| = 2.5\sqrt{2}\,[\text{u}]$

P est donc équidistant à A, B et C donc P est le centre du cercle circonscrit au triangle ABC.

c) $\sigma(\Delta ABC) = \frac{1}{2} \cdot |\det(\overrightarrow{AB}\,;\overrightarrow{AC})| = \frac{1}{2} \cdot \begin{vmatrix} 4 & 6 \\ -3 & 3 \end{vmatrix} = \frac{1}{2} \cdot [4 \cdot 3 - (-3) \cdot 6] = \frac{1}{2} \cdot 30 = 15\,[u^2]$

d) $* \ \overrightarrow{AB} \bullet \overrightarrow{AC} = 24 - 9 = 15$

$* \ \cos(\alpha) = \frac{\overrightarrow{AB} \bullet \overrightarrow{AC}}{||\overrightarrow{AB}|| \cdot ||\overrightarrow{AC}||} = \frac{15}{5 \cdot 3\sqrt{5}} = \frac{1}{\sqrt{5}} \Rightarrow \alpha = \arccos\left(\frac{1}{\sqrt{5}}\right) \cong \boxed{63.43^\circ}$

$* \ \overrightarrow{BA} \bullet \overrightarrow{BC} = -8 + 18 = 10$

$* \ \cos(\beta) = \frac{\overrightarrow{BA} \bullet \overrightarrow{BC}}{||\overrightarrow{AB}|| \cdot ||\overrightarrow{BC}||} = \frac{10}{5 \cdot 2\sqrt{10}} = \frac{1}{\sqrt{10}} \Rightarrow \beta = \arccos\left(\frac{1}{\sqrt{10}}\right) \cong \boxed{71.57^\circ}$

$* \ \overrightarrow{CA} \bullet \overrightarrow{CB} = 12 + 18 = 30$

$* \ \cos(\gamma) = \dfrac{\overrightarrow{CA} \bullet \overrightarrow{CB}}{||\overrightarrow{AC}|| \cdot ||\overrightarrow{BC}||} = \dfrac{30}{6\sqrt{5 \cdot 10}} = \dfrac{30}{30\sqrt{2}} = \dfrac{1}{\sqrt{2}} \Rightarrow$

$\Rightarrow \gamma = \arccos\left(\dfrac{1}{\sqrt{2}}\right) = \boxed{45^\circ}$