

**Exercice 51.**

$$\text{a) } * \vec{a} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} ; \quad \vec{b} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$$

$$* \|\vec{a}\| = \sqrt{10}[u] ; \quad \|\vec{b}\| = \sqrt{45} = 3\sqrt{5}[u]$$

$$* \vec{a} \bullet \vec{b} = 18 - 3 = 15$$

$$* \cos(\alpha) = \frac{\vec{a} \bullet \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|} = \frac{15}{\sqrt{10} \cdot 3\sqrt{5}} = \frac{15}{15\sqrt{2}} \Rightarrow \alpha = \arccos\left(\frac{1}{\sqrt{2}}\right) = \boxed{45^\circ}$$

$$\text{b) } * \vec{a} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} ; \quad \vec{b} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$$

$$* \|\vec{a}\| = \sqrt{25} = 5[u] ; \quad \|\vec{b}\| = \sqrt{26}[u]$$

$$* \vec{a} \bullet \vec{b} = -15 - 4 = -19$$

$$* \cos(\alpha) = \frac{\vec{a} \bullet \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|} = \frac{-19}{5\sqrt{26}} \Rightarrow \alpha = \arccos\left(\frac{-19}{5\sqrt{26}}\right) \cong \boxed{138.18^\circ}$$

$$\text{c) } * \vec{a} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} ; \quad \vec{b} = \begin{pmatrix} 6 \\ 15 \end{pmatrix}$$

$$* \|\vec{a}\| = \sqrt{29}[u] ; \quad \|\vec{b}\| = \sqrt{261}[u]$$

$$* \vec{a} \bullet \vec{b} = 30 - 30 = 0 \Rightarrow \vec{a} \perp \vec{b} \Rightarrow \alpha = \boxed{90^\circ}$$

**Exercice 52.**

$$\text{a) } * \vec{a} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} ; \quad \vec{b} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

$$* \|\vec{a}\| = \sqrt{9} = 3[u] \quad ; \quad \|\vec{b}\| = \sqrt{9} = 3[u]$$

$$* \vec{a} \bullet \vec{b} = 2 - 2 + 4 = 4$$

$$* \cos(\alpha) = \frac{\vec{a} \bullet \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|} = \frac{4}{3 \cdot 3} = \frac{4}{9} \Rightarrow \alpha = \arccos\left(\frac{4}{9}\right) \cong \boxed{63.61^\circ}$$

$$\text{b) } * \vec{a} = \begin{pmatrix} 3 \\ -4 \\ -1 \end{pmatrix} ; \quad \vec{b} = \begin{pmatrix} -2 \\ 3 \\ -2 \end{pmatrix}$$

$$* \|\vec{a}\| = \sqrt{26}[u] \quad ; \quad \|\vec{b}\| = \sqrt{17}[u]$$

$$* \vec{a} \bullet \vec{b} = -6 - 12 + 2 = -16$$

$$* \cos(\alpha) = \frac{\vec{a} \bullet \vec{b}}{\|\vec{a}\| \cdot \|\vec{b}\|} = \frac{-16}{\sqrt{26} \cdot \sqrt{17}} \Rightarrow \alpha = \arccos\left(\frac{-16}{\sqrt{442}}\right) \cong \boxed{139.56^\circ}$$

$$\text{c) } * \vec{a} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} ; \quad \vec{b} = \begin{pmatrix} 6 \\ 6 \\ -2 \end{pmatrix}$$

$$* \|\vec{a}\| = \sqrt{14}[u] \quad ; \quad \|\vec{b}\| = \sqrt{76} = 2\sqrt{19}[u]$$

$$* \vec{a} \bullet \vec{b} = 12 - 6 - 6 = 0 \Rightarrow \vec{a} \perp \vec{b} \Rightarrow \alpha = \boxed{90^\circ}$$