## Exercice 32.

$*$ On a $\overrightarrow{AB} = \begin{pmatrix} -4 \\ -4 \\ 3 \end{pmatrix}$

$*$ $P \in Ox \Rightarrow P(x\,;0\,;0) \Rightarrow \overrightarrow{AP} = \begin{pmatrix} x+2 \\ -3 \\ 2 \end{pmatrix} \quad ; \quad \overrightarrow{BP} = \begin{pmatrix} x+6 \\ 1 \\ -1 \end{pmatrix}$

$*$ $APB$ rectangle en $A \iff \overrightarrow{AB} \perp \overrightarrow{AP} \iff \overrightarrow{AB} \bullet \overrightarrow{AP} = 0 \iff$

$$\iff \begin{pmatrix} -4 \\ -4 \\ 3 \end{pmatrix} \bullet \begin{pmatrix} x+2 \\ -3 \\ 2 \end{pmatrix} = 0 \iff (-4)\cdot(x+2) + (-4)\cdot(-3) + 3\cdot 2 = 0 \iff$$

$$\iff -4x - 8 + 12 + 6 = 0 \iff 4x = 10 \iff x_1 = 5/2 \Rightarrow \boxed{P_1(5/2\,;0\,;0)}$$

$*$ $APB$ rectangle en $B \iff \overrightarrow{AB} \perp \overrightarrow{BP} \iff \overrightarrow{AB} \bullet \overrightarrow{BP} = 0 \iff$

$$\iff \begin{pmatrix} -4 \\ -4 \\ 3 \end{pmatrix} \bullet \begin{pmatrix} x+6 \\ 1 \\ -1 \end{pmatrix} = 0 \iff (-4)\cdot(x+6) + (-4)\cdot 1 + 3\cdot(-1) = 0 \iff$$

$$\iff -4x - 24 - 4 - 3 = 0 \iff 4x = -31 \iff x_2 = -31/4 \Rightarrow \boxed{P_2(-31/4\,;0\,;0)}$$

$*$ $APB$ rectangle en $P \iff \overrightarrow{AP} \perp \overrightarrow{BP} \iff \overrightarrow{AP} \bullet \overrightarrow{BP} = 0 \iff$

$$\iff \begin{pmatrix} x+2 \\ -3 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} x+6 \\ 1 \\ -1 \end{pmatrix} = 0 \iff (x+2)\cdot(x+6) + (-3)\cdot 1 + 2\cdot(-1) = 0 \iff$$

$$\iff x^2 + 8x + 12 - 3 - 2 = 0 \iff x^2 + 8x + 7 = 0 \iff (x+7)(x+1) = 0 \iff$$

$$\iff x_3 = -7 \text{ ou } x_4 = -1 \Rightarrow \boxed{P_3(-7\,;0\,;0) \text{ ou } P_4(-1\,;0\,;0)}$$

## Exercice 33.

$*$ On a $\overrightarrow{AB} = \begin{pmatrix} 1 \\ -6 \end{pmatrix}$

$*$ Soit $C(x\,;y) \Rightarrow \overrightarrow{AC} = \begin{pmatrix} x-2 \\ y-1 \end{pmatrix}$ ; $\overrightarrow{BC} = \begin{pmatrix} x-3 \\ y+5 \end{pmatrix}$

a) $ABCD$ est un carré :

1) $\overrightarrow{AB} \perp \overrightarrow{BC} \iff \overrightarrow{AB} \bullet \overrightarrow{BC} = 0 \iff$

$$\iff \begin{pmatrix} 1 \\ -6 \end{pmatrix} \bullet \begin{pmatrix} x-3 \\ y+5 \end{pmatrix} = 0 \iff 1 \cdot (x-3) + (-6) \cdot (y+5) = 0 \iff$$

$$\iff x - 3 - 6y - 30 = 0 \iff x - 6y - 33 = 0$$

2) $||\overrightarrow{AB}|| = ||\overrightarrow{BC}|| \iff$

$$\iff \sqrt{1^2 + (-6)^2} = \sqrt{(x-3)^2 + (y+5)^2} \overset{()^2}{\Rightarrow}$$

$$\overset{()^2}{\Rightarrow} 37 = x^2 - 6x + 9 + y^2 + 10y + 25 \iff$$

$$\iff x^2 - 6x + y^2 + 10y - 3 = 0$$

3) on va résoudre le système suivant par substitution :

$$\begin{cases} x - 6y - 33 = 0 \\ x^2 - 6x + y^2 + 10y - 3 = 0 \end{cases} \iff \begin{cases} x = 6y + 33 \\ (6y+33)^2 - 6(6y+33) + y^2 + 10y - 3 = 0 \end{cases} \Rightarrow$$

$$\Rightarrow 36y^2 + 396y + 1089 - 36y - 198 + y^2 + 10y - 3 = 0 \iff 37y^2 + 370y + 888 = 0 \iff$$

$$\iff 37(y^2 + 10y + 24) = 0 \iff 37(y+6)(y+4)) = 0 \iff y_1 = -6 \text{ ou } y_2 = -4$$

et $x_1 = -3$ ou $x_2 = 9 \Rightarrow$ $\boxed{C_1(-3\,;-6) \text{ ou } C_2(9\,;-4)}$

4) Soit $D_1(u_1\,;v_1)$

$$\overrightarrow{BA} = \overrightarrow{C_1D_1} \iff \begin{pmatrix} -1 \\ 6 \end{pmatrix} = \begin{pmatrix} u_1+3 \\ v_1+6 \end{pmatrix} \Rightarrow u_1 = -4 \text{ et } v_1 = 0 \Rightarrow \boxed{D_1(-4\,;0)}$$

5) Soit $D_2(u_2\,;v_2)$

$$\overrightarrow{BA} = \overrightarrow{C_2D_2} \iff \begin{pmatrix} -1 \\ 6 \end{pmatrix} = \begin{pmatrix} u_2-9 \\ v_2+4 \end{pmatrix} \Rightarrow u_2 = 8 \text{ et } v_2 = 2 \Rightarrow \boxed{D_2(8\,;2)}$$

ou autre méthode ...

b) $ACBD$ est un carré :

1) $\overrightarrow{AC} \perp \overrightarrow{BC} \iff \overrightarrow{AC} \bullet \overrightarrow{BC} = 0 \iff$

$$\iff \begin{pmatrix} x-2 \\ y-1 \end{pmatrix} \bullet \begin{pmatrix} x-3 \\ y+5 \end{pmatrix} = 0 \iff (x-2)\cdot(x-3) + (y-1)\cdot(y+5) = 0 \iff$$

$$\iff x^2 - 5x + 6 + y^2 + 4y - 5 = 0 \iff x^2 - 5x + y^2 + 4y + 1 = 0$$

2) $||\overrightarrow{AC}|| = ||\overrightarrow{BC}|| \iff$

$$\iff \sqrt{(x-2)^2 + (y-1)^2} = \sqrt{(x-3)^2 + (y+5)^2} \overset{()^2}{\Rightarrow}$$

$$\overset{()^2}{\Rightarrow} x^2 - 4x + 4 + y^2 - 2y + 1 = x^2 - 6x + 9 + y^2 + 10y + 25 \iff$$

$$\iff 2x - 12y - 29 = 0$$

3) on va résoudre le système suivant par substitution :

$$\begin{cases} x^2 - 5x + y^2 + 4y + 1 = 0 \\ 2x - 12y - 29 = 0 \end{cases} \iff \begin{cases} \left(\dfrac{12y+29}{2}\right)^2 - 5\left(\dfrac{12y+29}{2}\right) + y^2 + 4y + 1 = 0 \\ x = \dfrac{12y+29}{2} \end{cases}$$

$$\Rightarrow \frac{144y^2 + 696y + 841}{4} - \frac{60y + 145}{2} + y^2 + 4y + 1 = 0 \overset{\cdot 4}{\iff}$$

$$\iff 144y^2 + 696y + 841 - 120y - 290 + 4y^2 + 16y + 4 = 0 \iff 148y^2 + 592y + 555 = 0 \iff$$

$$\iff 37(4y^2 + 16y + 15) = 0 \iff 37(2y+5)(2y+3)) = 0 \iff y_1 = -5/2 \text{ ou } y_2 = -3/2$$

et $x_1 = -1/2$ ou $x_2 = 11/2 \Rightarrow \boxed{C(-1/2\,;-5/2) \text{ ou } D(11/2\,;-3/2)}$

ou autre méthode ...

## Exercice 34.

$*$ On a $\overrightarrow{BC} = \begin{pmatrix} -2 \\ -6 \end{pmatrix}$

$*$ Soit $A(x\,;y) \Rightarrow \overrightarrow{BA} = \begin{pmatrix} x-3 \\ y-4 \end{pmatrix} \quad ; \quad \overrightarrow{CA} = \begin{pmatrix} x-1 \\ y+2 \end{pmatrix}$

$ABC$ est un triangle rectangle et isocèle en $A$ :

1) $\overrightarrow{BA} \perp \overrightarrow{CA} \iff \overrightarrow{BA} \bullet \overrightarrow{CA} = 0 \iff$

$$\iff \begin{pmatrix} x-3 \\ y-4 \end{pmatrix} \bullet \begin{pmatrix} x-1 \\ y+2 \end{pmatrix} = 0 \iff (x-3)\cdot(x-1) + (y-4)\cdot(y+2) = 0 \iff$$

$$\iff x^2 - 4x + 3 + y^2 - 2y - 8 = 0 \iff x^2 - 4x + y^2 - 2y - 5 = 0$$

2) $||\overrightarrow{BA}|| = ||\overrightarrow{CA}|| \iff$

$$\iff \sqrt{(x-3)^2 + (y-4)^2} = \sqrt{(x-1)^2 + (y+2)^2} \overset{()^2}{\Rightarrow}$$

$$\overset{()^2}{\Rightarrow} x^2 - 6x + 9 + y^2 - 8y + 16 = x^2 - 2x + 1 + y^2 + 4y + 4 \iff$$

$$\iff 4x + 12y - 20 = 0 \iff x + 3y - 5 = 0$$

3) on va résoudre le système suivant par substitution :

$$\begin{cases} x^2 - 4x + y^2 - 2y - 5 = 0 \\ x + 3y - 5 = 0 \end{cases} \iff \begin{cases} (-3y+5)^2 - 4(-3y+5) + y^2 - 2y - 5 = 0 \\ x = -3y + 5 \end{cases} \Rightarrow$$

$$\Rightarrow 9y^2 - 30y + 25 + 12y - 20 + y^2 - 2y - 5 = 0 \iff 10y^2 - 20y = 0 \iff$$

$$\iff 10y(y-2) = 0 \iff y_1 = 0 \text{ ou } y_2 = 2$$

$$\text{et } x_1 = 5 \text{ ou } x_2 = -1 \Rightarrow \boxed{A_1(5\,;0) \text{ ou } A_2(-1\,;2)}$$

ou autre méthode ...

## Exercice 35.

$*$ On a $\overrightarrow{AB} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$

$*$ $C(\lambda\,;\lambda) \Rightarrow \overrightarrow{AC} = \begin{pmatrix} \lambda+2 \\ \lambda-4 \end{pmatrix}$ ; $\overrightarrow{BC} = \begin{pmatrix} \lambda-1 \\ \lambda+2 \end{pmatrix}$

1) $ABC$ est un triangle rectangle en $A$ :

$$\overrightarrow{AB} \perp \overrightarrow{AC} \iff \overrightarrow{AB} \bullet \overrightarrow{AC} = 0 \iff \begin{pmatrix} 3 \\ -6 \end{pmatrix} \bullet \begin{pmatrix} \lambda+2 \\ \lambda-4 \end{pmatrix} = 0 \iff$$

$$\iff 3 \cdot (\lambda+2) + (-6) \cdot (\lambda-4) = 0 \iff 3\lambda + 6 - 6\lambda + 24 = 0 \iff$$

$$\iff -3\lambda + 30 = 0 \iff 3\lambda = 30 \iff \boxed{\lambda_1 = 10}$$

Remarque : $||\overrightarrow{AB}|| = \sqrt{45} = 3\sqrt{5}$ ; $||\overrightarrow{AC}|| = ||\begin{pmatrix} 12 \\ 6 \end{pmatrix}|| = \sqrt{180} = 6\sqrt{5}$

2) $ABC$ est un triangle rectangle en $B$ :

$$\overrightarrow{AB} \perp \overrightarrow{BC} \iff \overrightarrow{AB} \bullet \overrightarrow{BC} = 0 \iff \begin{pmatrix} 3 \\ -6 \end{pmatrix} \bullet \begin{pmatrix} \lambda-1 \\ \lambda+2 \end{pmatrix} = 0 \iff$$

$$\iff 3 \cdot (\lambda-1) + (-6) \cdot (\lambda+2) = 0 \iff 3\lambda - 3 - 6\lambda - 12 = 0 \iff$$

$$\iff -3\lambda - 15 = 0 \iff 3\lambda = -15 \iff \boxed{\lambda_2 = -5}$$

Remarque : $||\overrightarrow{AB}|| = \sqrt{45} = 3\sqrt{5}$ ; $||\overrightarrow{BC}|| = ||\begin{pmatrix} -6 \\ -3 \end{pmatrix}|| = \sqrt{45} = 3\sqrt{5}$

Pour $\lambda_2 = -5$, $ABC$ sera rectangle et isocèle en $B$.

3) $ABC$ est un triangle rectangle en $C$ :

$$\overrightarrow{AC} \perp \overrightarrow{BC} \iff \overrightarrow{AC} \bullet \overrightarrow{BC} = 0 \iff \begin{pmatrix} \lambda+2 \\ \lambda-4 \end{pmatrix} \bullet \begin{pmatrix} \lambda-1 \\ \lambda+2 \end{pmatrix} = 0 \iff$$

$$\iff (\lambda+2) \cdot (\lambda-1) + (\lambda-4) \cdot (\lambda+2) = 0 \iff \lambda^2 + \lambda - 2 + \lambda^2 - 2\lambda - 8 = 0 \iff$$

$$\iff 2\lambda^2 - \lambda - 10 = 0 \iff (2\lambda-5)(\lambda+2) = 0 \iff \boxed{\lambda_3 = 5/2 \text{ ou } \lambda_4 = -2}$$

Remarques :

- $\lambda_3 = 5/2$ : $||\overrightarrow{AC}|| = ||\begin{pmatrix} 9/2 \\ -3/2 \end{pmatrix}|| = \sqrt{90/4} = 3\sqrt{10}/2$

$||\overrightarrow{BC}|| = ||\begin{pmatrix} 3/2 \\ 9/2 \end{pmatrix}|| = 3\sqrt{10}/2 \Rightarrow ABC$ sera rectangle et isocèle en $C$.

- $\lambda_4 = -2$ : $||\overrightarrow{AC}|| = ||\begin{pmatrix} 0 \\ -6 \end{pmatrix}|| = \sqrt{36} = 6$ ; $||\overrightarrow{BC}|| = ||\begin{pmatrix} -3 \\ 0 \end{pmatrix}|| = \sqrt{9} = 3$