Exercice 24.

$\overrightarrow{b}$ est orthogonal à $\overrightarrow{a} \iff \overrightarrow{b} \bullet \overrightarrow{a} = 0 \iff \begin{pmatrix} 2 \\ \lambda \end{pmatrix} \bullet \begin{pmatrix} 3 \\ -8 \end{pmatrix} = 0 \iff$

$\iff 2 \cdot 3 + \lambda \cdot (-8) = 0 \iff 6 - 8\lambda = 0 \iff 8\lambda = 6 \iff \boxed{\lambda = 3/4}$

Exercice 27.

$*$ On pose $\overrightarrow{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ tel que $||\overrightarrow{v}|| = \sqrt{v_1^2 + v_2^2} = 15 \iff v_1^2 + v_2^2 = 225$

$*$ $\overrightarrow{v}$ est orthogonal à $\overrightarrow{a} \iff \overrightarrow{v} \bullet \overrightarrow{a} = 0 \iff \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ -3 \end{pmatrix} = 0 \iff 4 \cdot v_1 - 3 \cdot v_2 = 0$

$*$ on va résoudre le système suivant par substitution :

$$\begin{cases} v_1^2 + v_2^2 = 225 \\ 4v_1 - 3v_2 = 0 \end{cases} \iff \begin{cases} v_1^2 + v_2^2 = 225 \\ v_2 = \dfrac{4}{3}\, v_1 \end{cases} \Rightarrow$$

$$\Rightarrow v_1^2 + \frac{16}{9}\, v_1^2 = 225 \iff 9v_1^2 + 16v_1^2 = 2025 \iff 25v_1^2 - 2025 = 0 \iff$$

$$\iff 25(v_1^2 - 81) = 0 \iff 25(v_1 + 9)(v_1 - 9) = 0 \iff v_1 = -9 \text{ ou } v_1' = 9$$

$$\text{et } v_2 = -12 \text{ ou } v_2' = 12 \Rightarrow \boxed{\overrightarrow{v} = \begin{pmatrix} -9 \\ -12 \end{pmatrix} \text{ ou } \overrightarrow{v'} = \begin{pmatrix} 9 \\ 12 \end{pmatrix}}$$

ou autre méthode.

Exercice 28.

$*$ On pose $\overrightarrow{v} = \begin{pmatrix} 0 \\ v_2 \\ v_3 \end{pmatrix}$ tel que $||\overrightarrow{v}|| = \sqrt{0^2 + v_2^2 + v_3^2} = 1 \iff v_2^2 + v_3^2 = 1$

$*$ $\overrightarrow{v}$ est orthogonal à $\overrightarrow{a} \iff \overrightarrow{v} \bullet \overrightarrow{a} = 0 \iff \begin{pmatrix} 0 \\ v_2 \\ v_3 \end{pmatrix} \bullet \begin{pmatrix} 5 \\ -4 \\ 3 \end{pmatrix} = 0 \iff$

$\iff 0 \cdot 5 + (-4) \cdot v_2 + 3 \cdot v_3 = 0 \iff -4v_2 + 3v_3 = 0$

$*$ on va résoudre le système suivant par substitution :

$$\begin{cases} v_2^2 + v_3^2 = 1 \\ -4v_2 + 3v_3 = 0 \end{cases} \iff \begin{cases} v_2^2 + v_3^2 = 1 \\ v_3 = \dfrac{4}{3}\, v_2 \end{cases} \Rightarrow$$

$$\Rightarrow v_2^2 + \frac{16}{9}\, v_2^2 = 1 \iff 9v_2^2 + 16v_2^2 = 9 \iff 25v_2^2 - 9 = 0 \iff$$

$$\iff (5v_2 + 3)(5v_2 - 3) = 0 \iff v_2 = -\frac{3}{5} \text{ ou } v_2' = \frac{3}{5}$$

$$\text{et } v_3 = -\frac{4}{5} \text{ ou } v_2' = \frac{4}{5} \Rightarrow \boxed{\overrightarrow{v} = \begin{pmatrix} 0 \\ -3/5 \\ -4/5 \end{pmatrix} \text{ ou } \overrightarrow{v'} = \begin{pmatrix} 0 \\ 3/5 \\ 4/5 \end{pmatrix}}$$

ou autre méthode.

Exercice 29.

$*$ $\begin{pmatrix} 7 \\ s \\ t \end{pmatrix} \bullet \begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix} = 0 \iff 7 \cdot 4 + 3 \cdot s + 8 \cdot t = 0 \iff 3s + 8t = -28$

$*$ $\begin{pmatrix} 7 \\ s \\ t \end{pmatrix} \bullet \begin{pmatrix} -5 \\ 20 \\ 9 \end{pmatrix} = 0 \iff 7 \cdot (-5) + 20 \cdot s + 9 \cdot t = 0 \iff 20s + 9t = 35$

$*$ on va résoudre le système suivant par combinaison linéaire :

$$\begin{cases} 3s + 8t = -28 & | \cdot 9 \\ 20s + 9t = 35 & | \cdot (-8) \end{cases} \iff \begin{cases} 27s + 72t = -252 \\ -160s - 72t = -280 \end{cases} \Rightarrow$$

$$\Rightarrow -133s = -532 \Rightarrow \boxed{s = 4 \quad \text{et} \quad t = -5}$$