

Exercice 19.

$$* \vec{a} \bullet \vec{b} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} -1 \\ -5 \end{pmatrix} = 3 \cdot (-1) + 1 \cdot (-5) = -3 - 5 = \boxed{-8}$$

$$* \vec{a} \bullet \vec{c} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -3 \end{pmatrix} = 3 \cdot 1 + 1 \cdot (-3) = 3 - 3 = \boxed{0} \iff \vec{a} \perp \vec{c}$$

$$* \vec{a} \bullet \vec{e} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} -4 \\ 3 \end{pmatrix} = 3 \cdot (-4) + 1 \cdot 3 = -12 + 3 = \boxed{-9}$$

$$* \vec{b} \bullet \vec{a} = \begin{pmatrix} -1 \\ -5 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 1 \end{pmatrix} = (-1) \cdot 3 + (-5) \cdot 1 = -3 - 5 = \boxed{-8} = \vec{a} \bullet \vec{b}$$

$$* \vec{d} \bullet \vec{e} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} -4 \\ 3 \end{pmatrix} = 3 \cdot (-4) + 4 \cdot 3 = -12 + 12 = \boxed{0} \iff \vec{d} \perp \vec{e}$$

$$* \vec{c} \bullet \vec{d} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 4 \end{pmatrix} = 1 \cdot 3 + (-3) \cdot 4 = 3 - 12 = \boxed{-9}$$

$$* (\vec{a} + \vec{b}) \bullet (\vec{c} - \vec{d}) = \begin{pmatrix} 2 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} -2 \\ -7 \end{pmatrix} = 2 \cdot (-2) + (-4) \cdot (-7) = -4 + 28 = \boxed{24}$$

Exercice 20.

$$* \vec{a} \bullet \vec{b} = \begin{pmatrix} 8 \\ -9 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix} = 8 \cdot 5 + (-9) \cdot (-2) + 1 \cdot 3 = 40 + 18 + 3 = \boxed{61}$$

$$* \vec{b} \bullet \vec{a} = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix} \bullet \begin{pmatrix} 8 \\ -9 \\ 1 \end{pmatrix} = 5 \cdot 8 + (-2) \cdot (-9) + 3 \cdot 1 = 40 + 18 + 3 = \boxed{61} = \vec{a} \bullet \vec{b}$$

$$* \vec{a} \bullet \vec{c} = \begin{pmatrix} 8 \\ -9 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 0 \\ -8 \end{pmatrix} = 8 \cdot 1 + (-9) \cdot 0 + 1 \cdot (-8) = 8 + 0 - 8 = \boxed{0} \iff \vec{a} \perp \vec{c}$$

$$* \vec{c} \bullet \vec{a} = \begin{pmatrix} 1 \\ 0 \\ -8 \end{pmatrix} \bullet \begin{pmatrix} 8 \\ -9 \\ 1 \end{pmatrix} = 1 \cdot 8 + 0 \cdot (-9) + (-8) \cdot 1 = 8 + 0 - 8 = \boxed{0} \iff \vec{c} \perp \vec{a}$$

$$* \vec{b} \bullet \vec{c} = \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 0 \\ -8 \end{pmatrix} = 5 \cdot 1 + (-2) \cdot 0 + 3 \cdot (-8) = 5 + 0 - 24 = \boxed{-19}$$

$$* \vec{a} \bullet (\vec{b} + \vec{c}) = \begin{pmatrix} 8 \\ -9 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 6 \\ -2 \\ -5 \end{pmatrix} = 8 \cdot 6 + (-9) \cdot (-2) + 1 \cdot (-5) = 48 + 18 - 5 = \boxed{61}$$

$$* (\vec{c} - \vec{a}) \bullet (\vec{b} - \vec{a}) = \begin{pmatrix} -7 \\ 9 \\ -9 \end{pmatrix} \bullet \begin{pmatrix} -3 \\ 7 \\ 2 \end{pmatrix} = (-7) \cdot (-3) + 9 \cdot 7 + (-9) \cdot 2 = 21 + 63 - 18 = \boxed{66}$$

Exercice 21.

$$\begin{aligned}
 \text{a) } * \vec{a} \bullet \vec{d} &= \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = 1 \cdot 1 + 1 \cdot 2 + 1 \cdot (-3) = 1 + 2 - 3 = \boxed{0} \iff \vec{a} \perp \vec{d} \\
 * \vec{a} \bullet \vec{g} &= \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} = 1 \cdot 1 + 1 \cdot 1 + 1 \cdot (-2) = 1 + 1 - 2 = \boxed{0} \iff \vec{a} \perp \vec{g} \\
 * \vec{b} \bullet \vec{h} &= \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} = 2 \cdot (-2) + (-1) \cdot 0 + 4 \cdot 1 = -4 + 0 + 4 = \boxed{0} \iff \vec{b} \perp \vec{h} \\
 * \vec{d} \bullet \vec{i} &= \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} = 1 \cdot 4 + 2 \cdot 1 + (-3) \cdot 2 = 4 + 2 - 6 = \boxed{0} \iff \vec{d} \perp \vec{i}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } * (\vec{b} \bullet \vec{c}) \cdot \vec{a} &= \left(\begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right) \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 12 \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 12 \\ 12 \\ 12 \end{pmatrix} \\
 * (\vec{b} \bullet \vec{h}) \cdot \vec{c} &= \left(\begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} \right) \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 0 \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \\
 * (\vec{b} \bullet \vec{c}) \cdot \vec{i} &= \left(\begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right) \cdot \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} = 12 \cdot \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 48 \\ 12 \\ 24 \end{pmatrix} \\
 * (\vec{g} \bullet \vec{a}) \cdot \vec{d} &= \left(\begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right) \cdot \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = 0 \cdot \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \\
 * (\vec{a} \bullet \vec{b}) \cdot (\vec{d} \bullet \vec{a}) &= \left(\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \right) \cdot \left(\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right) = 2 \cdot 0 = \boxed{0}
 \end{aligned}$$

Exercice 22.

$$* \vec{AB} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} ; \vec{BC} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} ; \vec{CD} = \begin{pmatrix} -6 \\ -4 \end{pmatrix} ; \vec{AD} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

$$\Rightarrow \vec{AB} = \vec{DC} ; \vec{BC} = \vec{AD}$$

$$\Rightarrow \|\vec{AB}\| = \|\vec{CD}\| = \sqrt{52} = 2\sqrt{13} [\text{u}] \text{ et } \|\vec{BC}\| = \|\vec{AD}\| = \sqrt{13} [\text{u}]$$

$$* \vec{AB} \bullet \vec{BC} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ -3 \end{pmatrix} = 6 \cdot 2 + 4 \cdot (-3) = 12 - 12 = 0 \Rightarrow \vec{AB} \perp \vec{BC} \text{ etc ...}$$

$\Rightarrow ABCD$ est un quadrilatère avec deux paires de côtés parallèles et isométriques et l'angle en chaque sommet est droit donc c'est un rectangle.