Géométrie vectorielle affine

Série A

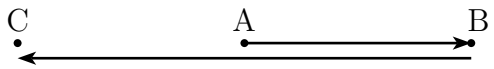
Exercice 1. (1+2=3 pts)

a) $\overrightarrow{BD} + \overrightarrow{AB} = \overrightarrow{AD}$

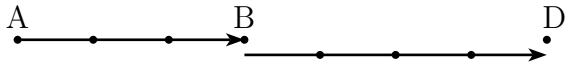
b) $\overrightarrow{AD} - \overrightarrow{BC} - \overrightarrow{AB} = \overrightarrow{CD}$

Exercice 2. (4 pts)

$\overrightarrow{BC} = (-2) \cdot \overrightarrow{AB}$



$\overrightarrow{BD} = \frac{4}{3} \cdot \overrightarrow{AB}$



Exercice 3. (3 pts)

$\vec{i}$ et $\vec{j}$ sont colinéaires $\Leftrightarrow \vec{i} = k \cdot \vec{j}$ avec $k \in \mathbb{R}$

$\Rightarrow \begin{pmatrix} 1/2 \\ -2/3 \end{pmatrix} = k \cdot \begin{pmatrix} -2/3 \\ 8/9 \end{pmatrix} \Rightarrow$

$\Rightarrow k = -\frac{3}{4}$

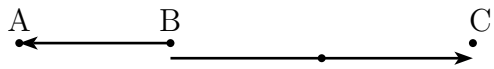
$\Rightarrow \vec{i}$ et $\vec{j}$ sont colinéaires

Série B

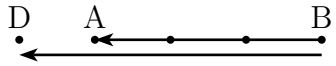
a) $\overrightarrow{BC} + \overrightarrow{AB} = \overrightarrow{AC}$

b) $\overrightarrow{AB} - \overrightarrow{CD} - \overrightarrow{AC} = \overrightarrow{DB}$

$\overrightarrow{BC} = (-2) \cdot \overrightarrow{BA}$



$\overrightarrow{BD} = \frac{4}{3} \cdot \overrightarrow{BA}$



$\vec{i}$ et $\vec{j}$ sont colinéaires $\Leftrightarrow \vec{i} = k \cdot \vec{j}$ avec $k \in \mathbb{R}$

$\Rightarrow \begin{pmatrix} 1/2 \\ -2/3 \end{pmatrix} = k \cdot \begin{pmatrix} -4/3 \\ 16/9 \end{pmatrix} \Rightarrow$

$\Rightarrow k = -\frac{3}{8}$

$\Rightarrow \vec{i}$ et $\vec{j}$ sont colinéaires

Exercice 4. (3 pts)

$\vec{u} = 5 \cdot \vec{t} - 2 \cdot \vec{r} + \frac{1}{3} \cdot \vec{s} =$

$= \begin{pmatrix} -5 \\ 15 \end{pmatrix} + \begin{pmatrix} -4 \\ 8 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix} = \begin{pmatrix} -7 \\ 19 \end{pmatrix}$

$\vec{u} = 4 \cdot \vec{t} - 3 \cdot \vec{r} - \frac{1}{3} \cdot \vec{s} =$

$= \begin{pmatrix} -4 \\ 12 \end{pmatrix} + \begin{pmatrix} -6 \\ 12 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} -12 \\ 28 \end{pmatrix}$

Exercice 5. (3 pts)

comb. linéaire : $\vec{x} = k \cdot \vec{v} + m \cdot \vec{w}$ avec $k, m \in \mathbb{R}$

$\Rightarrow \begin{pmatrix} -2 \\ 8 \end{pmatrix} = k \cdot \begin{pmatrix} 4 \\ 2 \end{pmatrix} + m \cdot \begin{pmatrix} -1 \\ 3 \end{pmatrix} \Rightarrow$

$\Rightarrow \begin{cases} -2 = 4k - m \\ 8 = 2k + 3m \end{cases} \Rightarrow \begin{cases} k = 1/7 \\ m = 18/7 \end{cases} \Rightarrow$

$\Rightarrow \vec{x} = \frac{1}{7} \cdot \vec{v} + \frac{18}{7} \cdot \vec{w}$

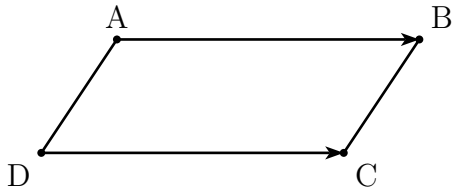
comb. linéaire : $\vec{x} = k \cdot \vec{v} + m \cdot \vec{w}$ avec $k, m \in \mathbb{R}$

$\Rightarrow \begin{pmatrix} -1 \\ 10 \end{pmatrix} = k \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix} + m \cdot \begin{pmatrix} -1 \\ 4 \end{pmatrix} \Rightarrow$

$\Rightarrow \begin{cases} -1 = 3k - m \\ 10 = 2k + 4m \end{cases} \Rightarrow \begin{cases} k = 3/7 \\ m = 16/7 \end{cases} \Rightarrow$

$\Rightarrow \vec{x} = \frac{3}{7} \cdot \vec{v} + \frac{16}{7} \cdot \vec{w}$

Exercice 6. (4 pts)



Posons $D(d_1\,;d_2)$

1) $\overrightarrow{AB} = \overrightarrow{DC} \Rightarrow \begin{pmatrix} 4 \\ 5 \end{pmatrix} = \begin{pmatrix} -1 - d_1 \\ 5 - d_2 \end{pmatrix}$

2) $\overrightarrow{BC} = \overrightarrow{AD} \Rightarrow \begin{pmatrix} -12 \\ 3 \end{pmatrix} = \begin{pmatrix} d_1 - 7 \\ d_2 + 3 \end{pmatrix}$

3) $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = \overrightarrow{OA} + \overrightarrow{BC} \Rightarrow$

$\Rightarrow \overrightarrow{OD} = \begin{pmatrix} 7 \\ -3 \end{pmatrix} + \begin{pmatrix} -12 \\ 3 \end{pmatrix} = \begin{pmatrix} -5 \\ 0 \end{pmatrix}$

$\Rightarrow D(-5\,;0)$

Posons $D(d_1\,;d_2)$

1) $\overrightarrow{AB} = \overrightarrow{DC} \Rightarrow \begin{pmatrix} -4 \\ -5 \end{pmatrix} = \begin{pmatrix} 1 - d_1 \\ -5 - d_2 \end{pmatrix}$

2) $\overrightarrow{BC} = \overrightarrow{AD} \Rightarrow \begin{pmatrix} 12 \\ -3 \end{pmatrix} = \begin{pmatrix} d_1 + 7 \\ d_2 - 3 \end{pmatrix}$

3) $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = \overrightarrow{OA} + \overrightarrow{BC} \Rightarrow$

$\Rightarrow \overrightarrow{OD} = \begin{pmatrix} -7 \\ 3 \end{pmatrix} + \begin{pmatrix} 12 \\ -3 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$

$\Rightarrow D(5\,;0)$
