Analyse - §6 : Fonction logarithme et fonction exponentielle

Exercice 1. (1.5+2+2.5+1=7 pts)

Série A

$$f(x) = \ln\left(\frac{x+4}{2x-1}\right)$$

a) • $\frac{x+4}{2x-1} > 0$

x		-4		0.5	
sgn $\frac{x+4}{2x-1}$	$+$	0	$-$	$\|$	$+$

$\Rightarrow ED_f =]-\infty\,;-4\,[\,\cup\,]\,\frac{1}{2}\,;+\infty\,[$

b) • $f(x) = 0 \iff \frac{x+4}{2x-1} = 1 \Rightarrow Z_f = \{5\}$

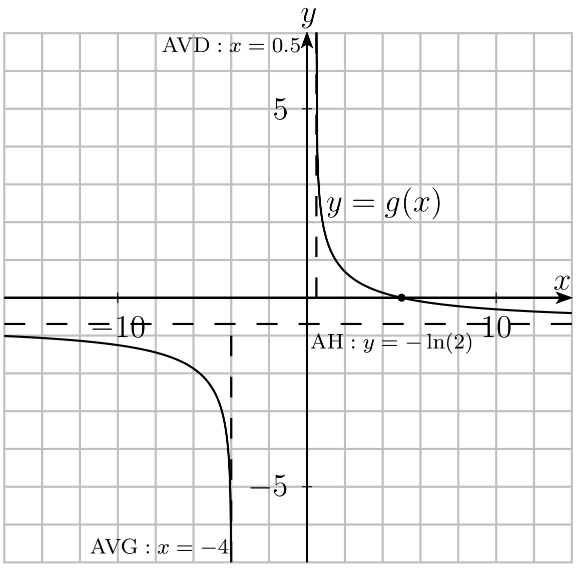
x		-4		0.5		5	
sgn(f)	$-$	$\|$	//////////	$\|$	$+$	0	$-$

c) • $\lim\limits_{\substack{x\to -4 \\ <}} f(x) = -\infty \Rightarrow$ AVG : $x = -4$

• $\lim\limits_{\substack{x\to 0.5 \\ >}} f(x) = +\infty \Rightarrow$ AVD : $x = 0.5$

• $\lim\limits_{x\to\pm\infty} f(x) = \ln(\frac{1}{2}) \Rightarrow$ AH : $y = -\ln(2)$

d)



Série B

$$f(x) = \ln\left(\frac{x-4}{2x+1}\right)$$

• $\frac{x-4}{2x+1} > 0$

x		-0.5		4	
sgn $\frac{x-4}{2x+1}$	$+$	$\|$	$-$	0	$+$

$\Rightarrow ED_f =]-\infty\,;-\frac{1}{2}\,[\,\cup\,]\,4\,;+\infty\,[$

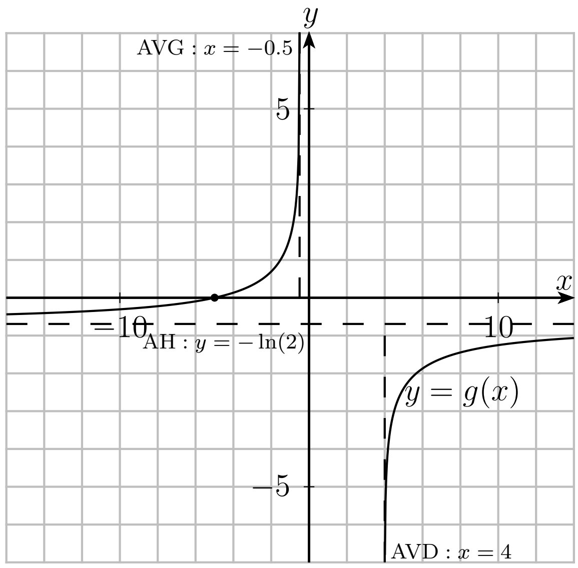
• $f(x) = 0 \iff \frac{x-4}{2x+1} = 1 \Rightarrow Z_f = \{-5\}$

x		-5		-0.5		4	
sgn(f)	$-$	0	$+$	$\|$	//////////	$\|$	$-$

• $\lim\limits_{\substack{x\to -0.5 \\ <}} f(x) = +\infty \Rightarrow$ AVG : $x = -0.5$

• $\lim\limits_{\substack{x\to 4 \\ >}} f(x) = -\infty \Rightarrow$ AVD : $x = 4$

• $\lim\limits_{x\to\pm\infty} f(x) = \ln(\frac{1}{2}) \Rightarrow$ AH : $y = -\ln(2)$



Exercice 2. (1+2.5+1.5+2+1=8 pts)

$g(x) = e^{\frac{x^2}{4(x+2)}}$

a) • $ED_g = \mathbb{R} \setminus \{-2\}$

• $g(x) = 0 \Rightarrow Z_g = \emptyset$

x		-2	
sgn(g)	$+$	$\|$	$+$

b) • $\lim\limits_{\substack{x\to -2 \\ <}} g(x) = "e^{\frac{4}{0^-}}" = 0 \Rightarrow$ pt. limite $(-2\,;0)$

• $\lim\limits_{\substack{x\to -2 \\ >}} g(x) = "e^{\frac{4}{0^+}}" = +\infty \Rightarrow$ AVD : $x = -2$

• $\lim\limits_{x\to -\infty} g(x) = "e^{-\infty}" = 0 \ \Rightarrow$ AHG : $y = 0$

• $\lim\limits_{x\to +\infty} g(x) = "e^{+\infty}" = +\infty$ (pas d'AHD)

c) $g'(x) = e^{\frac{x^2}{4(x+2)}} \cdot \dfrac{2x\cdot(4x+8) - x^2\cdot 4}{16(x+2)^2} =$

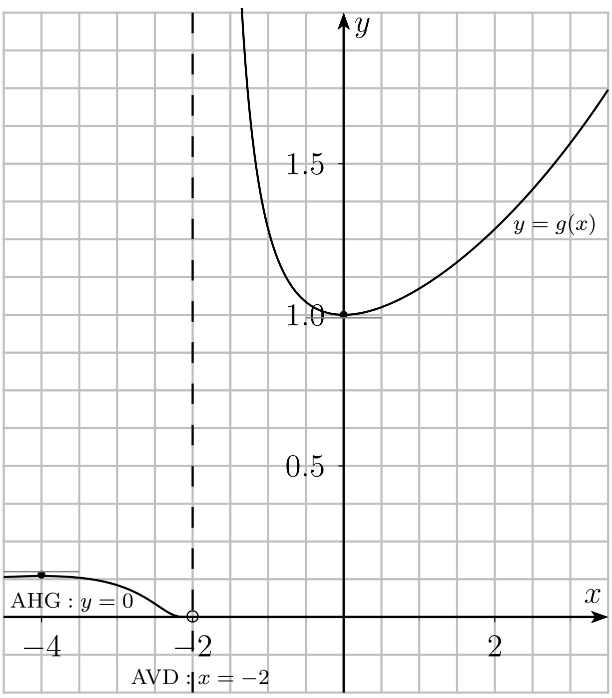
$= e^{\frac{x^2}{4(x+2)}} \cdot \dfrac{4(x^2+4x)}{16(x+2)^2} = e^{\frac{x^2}{4(x+2)}} \cdot \dfrac{x(x+4)}{4(x+2)^2}$

d) • $Z_{g'} = \{-4\,;0\}$

x		-4		-2		0	
sgn(g')	$+$	0	$-$	$\|$	$-$	0	$+$
var. de g	↗	max	↘	AV	↘	min	↗

max$(-4\,;e^{-2})$; min$(0\,;1)$

e)



$g(x) = e^{\frac{x^2}{4(x-2)}}$

• $ED_g = \mathbb{R} \setminus \{2\}$

• $g(x) = 0 \Rightarrow Z_g = \emptyset$

x		2	
sgn(g)	$+$	$\|$	$+$

• $\lim\limits_{\substack{x\to 2 \\ <}} g(x) = "e^{\frac{4}{0^-}}" = 0 \Rightarrow$ pt. limite $(2\,;0)$

• $\lim\limits_{\substack{x\to 2 \\ >}} g(x) = "e^{\frac{4}{0^+}}" = +\infty \Rightarrow$ AVD : $x = 2$

• $\lim\limits_{x\to -\infty} g(x) = "e^{-\infty}" = 0 \ \Rightarrow$ AHG : $y = 0$

• $\lim\limits_{x\to +\infty} g(x) = "e^{+\infty}" = +\infty$ (pas d'AHD)

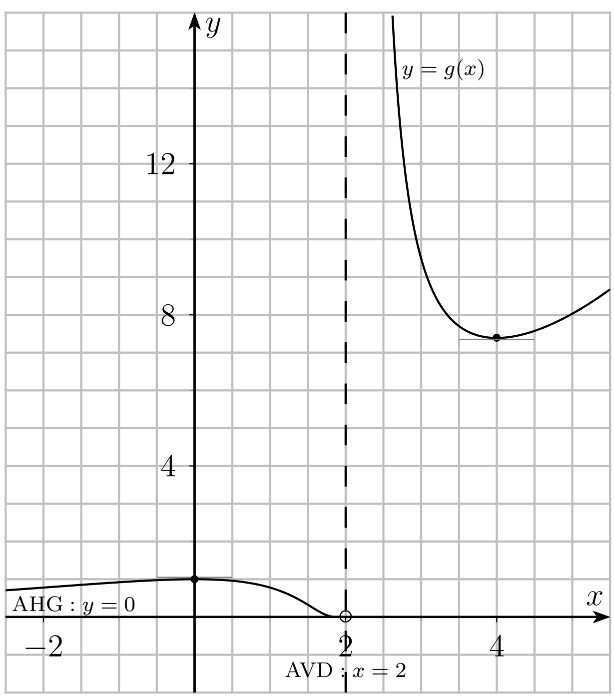
$g'(x) = e^{\frac{x^2}{4(x-2)}} \cdot \dfrac{2x\cdot(4x-8) - x^2\cdot 4}{16(x-2)^2} =$

$= e^{\frac{x^2}{4(x-2)}} \cdot \dfrac{4(x^2-4x)}{16(x-2)^2} = e^{\frac{x^2}{4(x-2)}} \cdot \dfrac{x(x-4)}{4(x-2)^2}$

• $Z_{g'} = \{0\,;4\}$

x		0		2		4	
sgn(g')	$+$	0	$-$	$\|$	$-$	0	$+$
var. de g	↗	max	↘	AV	↘	min	↗

max$(0\,;1)$; min$(4\,;e^2)$



Exercice 3. (3+2=5 pts)

a) $\int (3x+1)\,\mathrm{e}^{2x}\,dx =$

$f(x)=3x+1$	$g'(x)\,dx=\mathrm{e}^{2x}\,dx$
$f'(x)dx=3\,dx$	$g(x)=\frac{1}{2}\,\mathrm{e}^{2x}$

$$= \left[(3x+1)\frac{1}{2}\,\mathrm{e}^{2x}\right] - \int \frac{3}{2}\,\mathrm{e}^{2x}\,dx =$$

$$= \frac{3}{2}x\cdot\mathrm{e}^{2x} + \frac{1}{2}\,\mathrm{e}^{2x} - \frac{3}{4}\,\mathrm{e}^{2x} + c =$$

$$= \frac{3}{2}x\cdot\mathrm{e}^{2x} - \frac{1}{4}\,\mathrm{e}^{2x} + c$$

b) $\int_1^2 \frac{x+1}{x^2+2x}\,dx = \frac{1}{2}\int_1^2 \frac{2x+2}{x^2+2x}\,dx =$

$$= \frac{1}{2}\int_1^2 \frac{(x^2+2x)'}{x^2+2x}\,dx =$$

$$= \frac{1}{2}\left[\,\ln|x^2+2x|\,\right]_1^2 = \frac{1}{2}[\ln(8)-\ln(3)] =$$

$$= \frac{1}{2}\ln\left(\frac{8}{3}\right) \cong 0.49\,[\mathrm{u}^2]$$

• aussi possible par changement de variable :

$u = x^2+2x \Rightarrow du = (2x+2)\,dx \Rightarrow$
$\Rightarrow \frac{1}{2}\,du = (x+1)\,dx$

$$\int_1^2 \frac{x+1}{x^2+2x}\,dx = \frac{1}{2}\int_3^8 \frac{1}{u}\,du =$$

$$= \frac{1}{2}\left[\,\ln|u|\,\right]_3^8 = \frac{1}{2}[\ln(8)-\ln(3)] =$$

$$= \frac{1}{2}\ln\left(\frac{8}{3}\right) \cong 0.49\,[\mathrm{u}^2]$$

$\int (2x+1)\,\mathrm{e}^{3x}\,dx =$

$f(x)=2x+1$	$g'(x)\,dx=\mathrm{e}^{3x}\,dx$
$f'(x)dx=2\,dx$	$g(x)=\frac{1}{3}\,\mathrm{e}^{3x}$

$$= \left[(2x+1)\frac{1}{3}\,\mathrm{e}^{3x}\right] - \int \frac{2}{3}\,\mathrm{e}^{3x}\,dx =$$

$$= \frac{2}{3}x\cdot\mathrm{e}^{3x} + \frac{1}{3}\,\mathrm{e}^{3x} - \frac{2}{9}\,\mathrm{e}^{3x} + c =$$

$$= \frac{2}{3}x\cdot\mathrm{e}^{3x} + \frac{1}{9}\,\mathrm{e}^{3x} + c$$

$\int_1^2 \frac{x+2}{x^2+4x}\,dx = \frac{1}{2}\int_1^2 \frac{2x+4}{x^2+4x}\,dx =$

$$= \frac{1}{2}\int_1^2 \frac{(x^2+4x)'}{x^2+4x}\,dx =$$

$$= \frac{1}{2}\left[\,\ln|x^2+4x|\,\right]_1^2 = \frac{1}{2}[\ln(12)-\ln(5)] =$$

$$= \frac{1}{2}\ln\left(\frac{12}{5}\right) \cong 0.44\,[\mathrm{u}^2]$$

• aussi possible par changement de variable :

$u = x^2+4x \Rightarrow du = (2x+4)\,dx \Rightarrow$
$\Rightarrow \frac{1}{2}\,du = (x+2)\,dx$

$$\int_1^2 \frac{x+2}{x^2+4x}\,dx = \frac{1}{2}\int_5^{12} \frac{1}{u}\,du =$$

$$= \frac{1}{2}\left[\,\ln|u|\,\right]_5^{12} = \frac{1}{2}[\ln(12)-\ln(5)] =$$

$$= \frac{1}{2}\ln\left(\frac{12}{5}\right) \cong 0.44\,[\mathrm{u}^2]$$