

Exercice 5.7.

a) • $\mu = 0$ et $\sigma = \sqrt{9} = 3$

• $X \sim \mathcal{N}(0; 9) \Rightarrow Z = \frac{X - 0}{3} \sim \mathcal{N}(0; 1)$

• $z_c = \frac{c - 0}{3} = \frac{c}{3}$

• $P(X \geq c) = 10\% \Rightarrow 1 - P(X \leq c) = 10\% \Rightarrow P(X \leq c) = 90\% \Rightarrow$
 $\Rightarrow P\left(Z \leq \frac{c}{3}\right) = 0.9 \Rightarrow \Phi\left(\frac{c}{3}\right) = 0.9 \Rightarrow \frac{c}{3} \cong 1.28 \Rightarrow \boxed{c \cong 3.84}$

b) • $\mu = 5$ et $\sigma = \sqrt{1} = 1$

• $X \sim \mathcal{N}(5; 1) \Rightarrow Z = \frac{X - 5}{1} \sim \mathcal{N}(0; 1)$

• $z_c = \frac{c - 5}{1} = c - 5$

• $P(X \leq c) = 30\% \Rightarrow P(Z \leq c - 5) = 0.3 \stackrel{\text{symétrie}}{\Rightarrow} P(Z \geq -(c - 5)) = 0.3 \Rightarrow$
 $\Rightarrow 1 - P(Z \leq -c + 5) = 0.3 \Rightarrow P(Z \leq -c + 5) = 0.7 \Rightarrow \Phi(-c + 5) = 0.7 \Rightarrow$
 $\Rightarrow -c + 5 \cong 0.52 \Rightarrow \boxed{c \cong 4.48}$

c) • $\mu = 100$ et $\sigma = \sqrt{100} = 10$

• $X \sim \mathcal{N}(100; 100) \Rightarrow Z = \frac{X - 100}{10} \sim \mathcal{N}(0; 1)$

• $z_c = \frac{c - 100}{10}$

• $P(X > c) = 80\% \Rightarrow P\left(Z > \frac{c - 100}{10}\right) = 0.8 \Rightarrow 1 - P\left(Z \leq \frac{c - 100}{10}\right) = 0.8 \Rightarrow$
 $\Rightarrow P\left(Z \leq \frac{c - 100}{10}\right) = 0.2 \stackrel{\text{symétrie}}{\Rightarrow} P\left(Z \geq -\frac{c - 100}{10}\right) = 0.2 \Rightarrow$
 $\Rightarrow 1 - P\left(Z \leq -\frac{c - 100}{10}\right) = 0.2 \Rightarrow P\left(Z \leq -\frac{c - 100}{10}\right) = 0.8 \Rightarrow$
 $\Rightarrow \Phi\left(-\frac{c - 100}{10}\right) = 0.8 \Rightarrow -\frac{c - 100}{10} \cong 0.84 \Rightarrow \boxed{c \cong 91.6}$

d) • $\mu = -6$ et $\sigma = \sqrt{4} = 2$

• $X \sim \mathcal{N}(-6; 4) \Rightarrow Z = \frac{X - (-6)}{2} \sim \mathcal{N}(0; 1)$

• $z_c = \frac{c + 6}{2}$

• $P(X < c) = 95\% \Rightarrow P\left(Z < \frac{c + 6}{2}\right) = 0.95 \Rightarrow \Phi\left(\frac{c + 6}{2}\right) = 0.95 \Rightarrow$
 $\Rightarrow \frac{c + 6}{2} \cong 1.645 \Rightarrow \boxed{c \cong -2.71}$

e) • $\mu = 0$ et $\sigma = \sqrt{16} = 4$

• $X \sim \mathcal{N}(0; 16) \Rightarrow Z = \frac{X - 0}{4} \sim \mathcal{N}(0; 1)$

• $z_c = \frac{c}{4}$

• $P(|X| \geq c) = 1\% \Rightarrow P\left(|Z| \geq \frac{c}{4}\right) = 1\% \Rightarrow P\left(Z \leq -\frac{c}{4}\right) + P\left(Z \geq \frac{c}{4}\right) = 1\% \Rightarrow$

symétrie $\Rightarrow 2 \cdot P\left(Z \geq \frac{c}{4}\right) = 1\% \Rightarrow 2 \cdot [1 - P\left(Z \leq \frac{c}{4}\right)] = 1\% \Rightarrow$

$\Rightarrow P\left(Z \leq \frac{c}{4}\right) = 0.995 \Rightarrow \Phi\left(\frac{c}{4}\right) = 0.995 \Rightarrow \frac{c}{4} \cong 2.575 \Rightarrow \boxed{c \cong 10.3}$

f) • $\mu = 0$ et $\sigma = \sqrt{20}$

• $X \sim \mathcal{N}(0; 20) \Rightarrow Z = \frac{X - 0}{\sqrt{20}} \sim \mathcal{N}(0; 1)$

• $z_c = \frac{c}{\sqrt{20}}$

• $P(-c < X < c) = 97\% \Rightarrow P\left(-\frac{c}{\sqrt{20}} < Z < \frac{c}{\sqrt{20}}\right) = 97\% \Rightarrow$

symétrie $\Rightarrow 2 \cdot P\left(Z < \frac{c}{\sqrt{20}}\right) - 1 = 0.97 \Rightarrow P\left(Z < \frac{c}{\sqrt{20}}\right) = 0.985 \Rightarrow$

$\Rightarrow \Phi\left(\frac{c}{\sqrt{20}}\right) = 0.985 \Rightarrow \frac{c}{\sqrt{20}} \cong 2.17 \Rightarrow \boxed{c \cong 9.7}$