Exercice 4.7.

a) Méthode 1 :

- $\overrightarrow{a} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \quad ; \quad \overrightarrow{b} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$
- $||\overrightarrow{a}|| = \sqrt{34}\,[u] \quad ; \quad ||\overrightarrow{b}|| = \sqrt{2}\,[u]$
- $\overrightarrow{a} \bullet \overrightarrow{b} = 5 + (-3) = 2$
- $\cos(\phi) = \dfrac{|\overrightarrow{a} \bullet \overrightarrow{b}|}{||\overrightarrow{a}|| \cdot ||\overrightarrow{b}||} = \dfrac{2}{\sqrt{34 \cdot 2}} \Rightarrow \phi = \arccos\left(\dfrac{2}{\sqrt{68}}\right) \cong \boxed{75.96^\circ}$

Méthode 2 :

- $\overrightarrow{n_a} = \begin{pmatrix} 3 \\ -5 \end{pmatrix} \quad ; \quad \overrightarrow{n_b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
- $||\overrightarrow{n_a}|| = \sqrt{34}\,[u] \quad ; \quad ||\overrightarrow{n_b}|| = \sqrt{2}\,[u]$
- $\overrightarrow{n_a} \bullet \overrightarrow{n_b} = 3 + (-5) = -2$
- $\cos(\phi) = \dfrac{|\overrightarrow{n_a} \bullet \overrightarrow{n_b}|}{||\overrightarrow{n_a}|| \cdot ||\overrightarrow{n_b}||} = \dfrac{|-2|}{\sqrt{34 \cdot 2}} \Rightarrow \phi = \arccos\left(\dfrac{2}{\sqrt{68}}\right) \cong \boxed{75.96^\circ}$

b) Méthode 1 :

- $\overrightarrow{a} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \quad ; \quad \overrightarrow{b} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$
- $||\overrightarrow{a}|| = \sqrt{13}\,[u] \quad ; \quad ||\overrightarrow{b}|| = \sqrt{5}\,[u]$
- $\overrightarrow{a} \bullet \overrightarrow{b} = -6 + (-2) = -8$
- $\cos(\phi) = \dfrac{|\overrightarrow{a} \bullet \overrightarrow{b}|}{||\overrightarrow{a}|| \cdot ||\overrightarrow{b}||} = \dfrac{|-8|}{\sqrt{13 \cdot 5}} \Rightarrow \phi = \arccos\left(\dfrac{8}{\sqrt{65}}\right) \cong \boxed{7.13^\circ}$

Méthode 2 :

- $\overrightarrow{n_a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad ; \quad \overrightarrow{n_b} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$
- $||\overrightarrow{n_a}|| = \sqrt{13}\,[u] \quad ; \quad ||\overrightarrow{n_b}|| = \sqrt{5}\,[u]$
- $\overrightarrow{n_a} \bullet \overrightarrow{n_b} = 2 + 6 = 8$
- $\cos(\phi) = \dfrac{|\overrightarrow{n_a} \bullet \overrightarrow{n_b}|}{||\overrightarrow{n_a}|| \cdot ||\overrightarrow{n_b}||} = \dfrac{8}{\sqrt{13 \cdot 5}} \Rightarrow \phi = \arccos\left(\dfrac{8}{\sqrt{65}}\right) \cong \boxed{7.13^\circ}$

c) Méthode 1 :

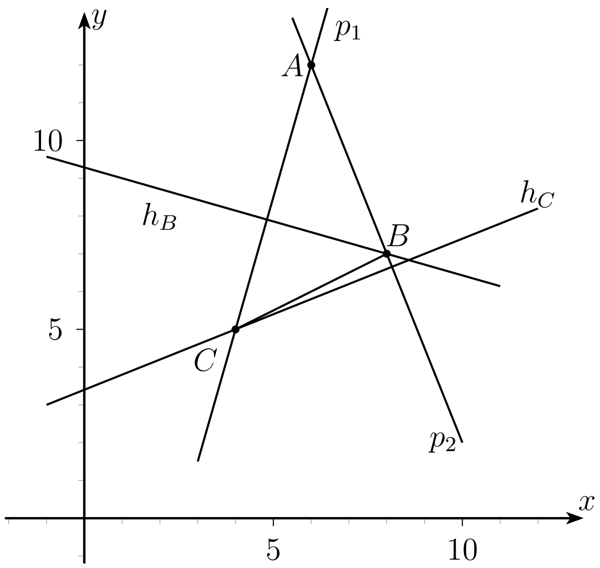
- $\overrightarrow{a} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad ; \quad \overrightarrow{b} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$
- $||\overrightarrow{a}|| = 1\,[u] \quad ; \quad ||\overrightarrow{b}|| = \sqrt{5}\,[u]$
- $\overrightarrow{a} \bullet \overrightarrow{b} = 0 + 1 = 1$
- $\cos(\phi) = \dfrac{|\overrightarrow{a} \bullet \overrightarrow{b}|}{||\overrightarrow{a}|| \cdot ||\overrightarrow{b}||} = \dfrac{1}{\sqrt{5}} \Rightarrow \phi = \arccos\left(\dfrac{1}{\sqrt{5}}\right) \cong \boxed{63.43^\circ}$

Méthode 2 :

- $\overrightarrow{n_a} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad ; \quad \overrightarrow{n_b} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$
- $||\overrightarrow{n_a}|| = 1\,[u] \quad ; \quad ||\overrightarrow{n_b}|| = \sqrt{5}\,[u]$
- $\overrightarrow{n_a} \bullet \overrightarrow{n_b} = 1 + 0 = 1$
- $\cos(\phi) = \dfrac{|\overrightarrow{n_a} \bullet \overrightarrow{n_b}|}{||\overrightarrow{n_a}|| \cdot ||\overrightarrow{n_b}||} = \dfrac{1}{\sqrt{5}} \Rightarrow \phi = \arccos\left(\dfrac{1}{\sqrt{5}}\right) \cong \boxed{63.43^\circ}$

Exercice 4.9.

Figure d'étude :



Marche à suivre :
1) $p_1 \perp h_B$ par A
2) $h_C \cap p_1 = C$
3) $p_2 \perp h_C$ par A
4) $h_B \cap p_2 = B$

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On donne $(h_B) : 2x + 7y - 65 = 0 \quad ; \quad (h_C) : 2x - 5y + 17 = 0$

1) $p_1 \perp h_B$ par $A \overset{\text{p.100}}{\Rightarrow} (p_1) : 7x - 2y + c = 0$ passe par $A(6\,;12)$:
$\Rightarrow 7 \cdot 6 - 2 \cdot 12 + c = 0 \iff 18 + c = 0 \iff c = -18$
$\Rightarrow (p_1) : 7x - 2y - 18 = 0$

2) $h_C \cap p_1 = C$:

$$\begin{cases} 2x - 5y + 17 = 0 \\ 7x - 2y - 18 = 0 \end{cases} \iff \begin{cases} 4x - 10y + 34 = 0 \\ -35x + 10y + 90 = 0 \end{cases} \iff \begin{cases} -31x + 124 = 0 \\ 7x - 2y - 18 = 0 \end{cases} \iff$$

$$\iff \begin{cases} x = 4 \\ y = 5 \end{cases} \Rightarrow \boxed{C(4\,;5)}$$

3) $p_2 \perp h_C$ par $A \overset{\text{p.100}}{\Rightarrow} (p_2) : -5x - 2y + c = 0$ passe par $A(6\,;12)$:
$\Rightarrow -5 \cdot 6 - 2 \cdot 12 + c = 0 \iff -54 + c = 0 \iff c = 54$
$\Rightarrow (p_2) : -5x - 2y + 54 = 0 \Rightarrow (p_2) : 5x + 2y - 54 = 0$

4) $h_B \cap p_2 = B$:

$$\begin{cases} 2x + 7y - 65 = 0 \\ -5x - 2y + 54 = 0 \end{cases} \iff \begin{cases} 4x + 14y - 130 = 0 \\ -35x - 14y + 378 = 0 \end{cases} \iff \begin{cases} -31x + 248 = 0 \\ 2x + 7y - 65 = 0 \end{cases} \iff$$

$$\iff \begin{cases} x = 8 \\ y = 7 \end{cases} \Rightarrow \boxed{B(8\,;7)}$$