

Exercice 4.3.

- $\vec{AB} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$; $\vec{BC} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$; $\vec{CD} = \begin{pmatrix} -3 \\ -9 \end{pmatrix}$; $\vec{AD} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$
- $\|\vec{AB}\| = \sqrt{50} [u]$; $\|\vec{BC}\| = \sqrt{90} [u]$; $\|\vec{CD}\| = \sqrt{90} [u]$; $\|\vec{AD}\| = \sqrt{50} [u]$
- $\vec{AC} = \begin{pmatrix} 10 \\ 10 \end{pmatrix}$; $\vec{BD} = \begin{pmatrix} 6 \\ -6 \end{pmatrix} \Rightarrow \vec{AC} \bullet \vec{BD} = 60 - 60 = 0 \Rightarrow \vec{AC} \perp \vec{BD}$
- par exemple :

$$\sigma(ABCD) = \sigma(ABC) + \sigma(ACD) = \frac{1}{2} \cdot |\det(\vec{AB}; \vec{AC})| + \frac{1}{2} \cdot |\det(\vec{AC}; \vec{AD})| =$$

$$= \frac{1}{2} \cdot \begin{vmatrix} 1 & 10 \\ 7 & 10 \end{vmatrix} + \frac{1}{2} \cdot \begin{vmatrix} 10 & 7 \\ 10 & 1 \end{vmatrix} = \frac{1}{2} \cdot |10 - 70| + \frac{1}{2} \cdot |10 - 70| = \frac{1}{2} \cdot |-60| + \frac{1}{2} \cdot |-60| =$$

$$= 30 + 30 = \boxed{60 [u^2]}$$

Exercice 4.4.

- $\vec{AB} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$; $\vec{BC} = \begin{pmatrix} -6 \\ -5 \end{pmatrix}$; $\vec{CD} = \begin{pmatrix} 5 \\ -5 \end{pmatrix}$; $\vec{AD} = \begin{pmatrix} -3 \\ -6 \end{pmatrix}$
- $\vec{AC} = \begin{pmatrix} -8 \\ -1 \end{pmatrix}$; $\vec{BD} = \begin{pmatrix} -1 \\ -10 \end{pmatrix}$
- $\sigma(ABCD) = \sigma(ABC) + \sigma(ACD) = \frac{1}{2} \cdot |\det(\vec{AB}; \vec{AC})| + \frac{1}{2} \cdot |\det(\vec{AC}; \vec{AD})| =$

$$= \frac{1}{2} \cdot \begin{vmatrix} -2 & -8 \\ 4 & -1 \end{vmatrix} + \frac{1}{2} \cdot \begin{vmatrix} -8 & -3 \\ -1 & -6 \end{vmatrix} = \frac{1}{2} \cdot |2 - (-32)| + \frac{1}{2} \cdot |48 - 3| = \frac{1}{2} \cdot |34| + \frac{1}{2} \cdot |45| =$$

$$= 17 + 22.5 = \boxed{39.5 [u^2]}$$

Exercice 4.5.

- On pose $B(x; y)$

- $\vec{AB} = \begin{pmatrix} x - 2 \\ y - 3 \end{pmatrix}$
- $\vec{PQ} = \begin{pmatrix} -6 \\ 12 \end{pmatrix} // \begin{pmatrix} 1 \\ -2 \end{pmatrix} \Rightarrow \vec{n}_{PQ} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \Rightarrow (PQ) : 2x + y + c = 0$ par $P(10; -3) \Rightarrow$
 $\Rightarrow 2 \cdot 10 + (-3) + c = 0 \iff 17 + c = 0 \iff c = -17 \Rightarrow (PQ) : 2x + y - 17 = 0$
- ΔABC est isocèle en A avec $\|\vec{AB}\| = \|\vec{AC}\| = 5$ et $B, C \in PQ$

$$\begin{cases} \sqrt{(x-2)^2 + (y-3)^2} = 5 \\ 2x + y - 17 = 0 \end{cases} \iff \begin{cases} (x-2)^2 + (y-3)^2 = 25 \\ y = -2x + 17 \end{cases} \Rightarrow$$

$$\Rightarrow x^2 - 4x + 4 + 4x^2 - 56x + 196 - 25 = 0 \iff 5x^2 - 60x + 175 = 0 \iff$$

$$\iff 5(x^2 - 12x + 35) = 0 \iff 5(x-5)(x-7) = 0 \iff x_1 = 5 \text{ ou } x_2 = 7$$

$$\text{et } y_1 = 7 \text{ ou } y_2 = 3 \Rightarrow \boxed{B(5; 7); C(7; 3)}$$