Exercice 4.1.

a) • $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$; $\overrightarrow{AC} = \begin{pmatrix} -6 \\ 8 \end{pmatrix}$; $\overrightarrow{BC} = \begin{pmatrix} -9 \\ 4 \end{pmatrix}$

• $||\overrightarrow{AB}|| = 5\,[u]$; $||\overrightarrow{AC}|| = 10\,[u]$; $||\overrightarrow{BC}|| = \sqrt{97}\,[u]$

• Le périmètre du $\Delta\,ABC = ||\overrightarrow{AB}|| + ||\overrightarrow{AC}|| + ||\overrightarrow{BC}|| = 5 + 10 + \sqrt{97} = \boxed{15 + \sqrt{97}\,[\text{u}]}$

b) • On pose $N(n_1\,;n_2)$

• $\overrightarrow{AB_u} = \dfrac{\overrightarrow{AB}}{||\overrightarrow{AB}||} = \begin{pmatrix} 3/5 \\ 4/5 \end{pmatrix}$

• $\overrightarrow{AN} = 2 \cdot \overrightarrow{AB_u} \Rightarrow \begin{pmatrix} n_1 - 2 \\ n_2 + 1 \end{pmatrix} = 2 \cdot \begin{pmatrix} 3/5 \\ 4/5 \end{pmatrix} \Rightarrow \boxed{N\left(\dfrac{16}{5}\,;\dfrac{3}{5}\right)}$

c) sans corrigé

Exercice 4.2.

a) ∘ $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$; $\overrightarrow{AC} = \begin{pmatrix} -4 \\ 8 \end{pmatrix}$; $\overrightarrow{BC} = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$

∘ $||\overrightarrow{AB}|| = \sqrt{26}\,[u]$; $||\overrightarrow{AC}|| = \sqrt{80} = 4\sqrt{5}\,[u]$; $||\overrightarrow{BC}|| = \sqrt{34}\,[u]$

∘ $\overrightarrow{AB} \bullet \overrightarrow{AC} = -4 + 40 = 36$; $\overrightarrow{BA} \bullet \overrightarrow{BC} = 5 - 15 = -10$; $\overrightarrow{CB} \bullet \overrightarrow{CA} = 20 + 24 = 44$

∘ $\cos(\alpha) = \dfrac{\overrightarrow{AB} \bullet \overrightarrow{AC}}{||\overrightarrow{AB}|| \cdot ||\overrightarrow{AC}||} = \dfrac{36}{\sqrt{26 \cdot 80}} \Rightarrow \alpha = \arccos\left(\dfrac{36}{\sqrt{26 \cdot 80}}\right) \cong \boxed{37.87°}$

$\cos(\beta) = \dfrac{\overrightarrow{BA} \bullet \overrightarrow{BC}}{||\overrightarrow{AB}|| \cdot ||\overrightarrow{BC}||} = \dfrac{-10}{\sqrt{26 \cdot 34}} \Rightarrow \beta = \arccos\left(-\dfrac{10}{\sqrt{26 \cdot 34}}\right) \cong \boxed{109.65°}$

$\cos(\gamma) = \dfrac{\overrightarrow{CB} \bullet \overrightarrow{CA}}{||\overrightarrow{BC}|| \cdot ||\overrightarrow{AC}||} = \dfrac{44}{\sqrt{34 \cdot 80}} \Rightarrow \gamma = \arccos\left(\dfrac{44}{\sqrt{34 \cdot 80}}\right) \cong \boxed{32.47°}$

b) $\sigma(ABC) = \dfrac{1}{2} \cdot |\det(\overrightarrow{AB}\,;\overrightarrow{AC})| = \dfrac{1}{2} \cdot \begin{vmatrix} 1 & -4 \\ 5 & 8 \end{vmatrix} = \dfrac{1}{2} \cdot |8 - (-20)| = \dfrac{1}{2} \cdot 28 = \boxed{14\,[\text{u}^2]}$