

Algèbre - chapitre 2 : Exponentielles et logarithmes

Série A

Série B

Exercice 1. (1+1+1+1=4 pts)

$$\text{a) } \log\left(\frac{2}{8 \cdot 25}\right) = \log\left(\frac{1}{10^2}\right) = -2$$

$$\text{b) } \log_a\left(\sqrt[3]{a^4}\right) = \log_a\left(a^{\frac{4}{3}}\right) = \frac{4}{3}$$

$$\text{c) } \log\left(10^n \cdot \frac{1}{10^{n+1}}\right) = \log(10^{-1}) = -1$$

$$\text{d) } \ln\left(\sqrt[3]{e^2}\right)^4 = \ln\left(e^{\frac{8}{3}}\right) = \frac{8}{3}$$

$$\log\left(\frac{5}{2 \cdot 25}\right) = \log\left(\frac{1}{10}\right) = -1$$

$$\log_a\left(\sqrt[3]{a^5}\right) = \log_a\left(a^{\frac{5}{3}}\right) = \frac{5}{3}$$

$$\log\left(10^n \cdot \frac{1}{10^{n+2}}\right) = \log(10^{-2}) = -2$$

$$\ln\left(\sqrt[3]{e^4}\right)^2 = \ln\left(e^{\frac{8}{3}}\right) = \frac{8}{3}$$

Exercice 2. (1+3=4 pts)

$$\text{a) } C(n) = 2'000 \cdot (1.005)^n$$

$$\text{b) } 2'000 \cdot (1 + 0.005)^n = 2'500 \Rightarrow$$

$$\Rightarrow (1.005)^n = \frac{5}{4} \Rightarrow$$

$$\Rightarrow n \cdot \ln(1.005) = \ln(5/4) \Rightarrow$$

$$\Rightarrow S = \left\{ \frac{\ln(5/4)}{\ln(1.005)} \right\} (\cong 44.74)$$

La durée sera d'environ 45 ans.

$$C(n) = 3'000 \cdot (1.004)^n$$

$$3'000 \cdot (1 + 0.004)^n = 3'500 \Rightarrow$$

$$\Rightarrow (1.004)^n = \frac{7}{6} \Rightarrow$$

$$\Rightarrow n \cdot \ln(1.004) = \ln(7/6) \Rightarrow$$

$$\Rightarrow S = \left\{ \frac{\ln(7/6)}{\ln(1.004)} \right\} (\cong 38.61)$$

La durée sera d'environ 39 ans.

Exercice 3. (1.5+1.5+1.5+2.5=7 pts)

$$\begin{aligned} \text{a) } 3^{x+1} \cdot 9^{-x} &= 81 \Rightarrow \\ \Rightarrow 3^{x+1} \cdot 3^{-2x} &= 3^4 \Rightarrow \\ \Rightarrow 3^{x+1-2x} &= 3^4 \Rightarrow \\ \Rightarrow x + 1 - 2x &= 4 \Rightarrow \end{aligned}$$

$$\Rightarrow S = \{-3\}$$

$$\begin{aligned} \text{b) } 4 \cdot e^{3x} &= 2\pi \Rightarrow \\ \Rightarrow e^{3x} &= \frac{\pi}{2} \Rightarrow \\ \Rightarrow 3x &= \ln(\pi/2) \Rightarrow \end{aligned}$$

$$\Rightarrow S = \left\{ \frac{\ln(\pi/2)}{3} \right\} (\cong 0.151)$$

$$\begin{aligned} \text{c) } 2 + 10^{2x-1} &= 5 \Rightarrow \\ \Rightarrow 10^{2x-1} &= 3 \Rightarrow \\ \Rightarrow 2x - 1 &= \log(3) \Rightarrow \\ \Rightarrow 2x &= 1 + \log(3) \Rightarrow \end{aligned}$$

$$\Rightarrow S = \left\{ \frac{1 + \log(3)}{2} \right\} (\cong 0.739)$$

$$\begin{aligned} \text{d) } \log(10x + 9) &= \log(x) + \log(100) \Rightarrow \\ \Rightarrow \log(10x + 9) &= \log(100x) \Rightarrow \\ \Rightarrow 10x + 9 &= 100x \Rightarrow \\ \Rightarrow x &= \frac{1}{10} \end{aligned}$$

OK équation définie

$$\Rightarrow S = \left\{ \frac{1}{10} \right\}$$

$$\begin{aligned} 2^{x+3} \cdot 8^{-x} &= 32 \Rightarrow \\ \Rightarrow 2^{x+3} \cdot 2^{-3x} &= 2^5 \Rightarrow \\ \Rightarrow 2^{x+3-3x} &= 2^5 \Rightarrow \\ \Rightarrow x + 3 - 3x &= 5 \Rightarrow \end{aligned}$$

$$\Rightarrow S = \{-1\}$$

$$\begin{aligned} 6 \cdot e^{2x} &= 3\pi \Rightarrow \\ \Rightarrow e^{2x} &= \frac{\pi}{2} \Rightarrow \\ \Rightarrow 2x &= \ln(\pi/2) \Rightarrow \end{aligned}$$

$$\Rightarrow S = \left\{ \frac{\ln(\pi/2)}{2} \right\} (\cong 0.226)$$

$$\begin{aligned} 3 + 10^{3x-1} &= 7 \Rightarrow \\ \Rightarrow 10^{3x-1} &= 4 \Rightarrow \\ \Rightarrow 3x - 1 &= \log(4) \Rightarrow \\ \Rightarrow 3x &= 1 + \log(4) \Rightarrow \end{aligned}$$

$$\Rightarrow S = \left\{ \frac{1 + \log(4)}{3} \right\} (\cong 0.534)$$

$$\begin{aligned} \log(30x + 7) &= \log(x) + \log(100) \Rightarrow \\ \Rightarrow \log(30x + 7) &= \log(100x) \Rightarrow \\ \Rightarrow 30x + 7 &= 100x \Rightarrow \\ \Rightarrow x &= \frac{1}{10} \end{aligned}$$

OK équation définie

$$\Rightarrow S = \left\{ \frac{1}{10} \right\}$$

Exercice 4. (1+4=5 pts)

a) $T(0) = 50 \cdot e^{-0.15 \cdot 0} + 25 = 50 + 25 = 75^\circ\text{C}$

b) VAR : $t =$ durée refroidissement café en min.

EQ : $50 \cdot e^{-0.15 \cdot t} + 25 = 40$

RES : $\Rightarrow 50 \cdot e^{-0.15 \cdot t} = 15$

$$\Rightarrow e^{-0.15 \cdot t} = \frac{3}{10}$$

$$\Rightarrow t = \frac{\ln(3/10)}{-0.15} (\cong 8.03 \text{ min})$$

SOL : La durée sera env. 8 minutes.

$T(0) = 45 \cdot e^{-0.15 \cdot 0} + 20 = 45 + 20 = 65^\circ\text{C}$

VAR : $t =$ durée refroidissement café en min.

EQ : $45 \cdot e^{-0.15 \cdot t} + 20 = 35$

RES : $\Rightarrow 45 \cdot e^{-0.15 \cdot t} = 15$

$$\Rightarrow e^{-0.15 \cdot t} = \frac{1}{3}$$

$$\Rightarrow t = \frac{\ln(1/3)}{-0.15} (\cong 7.32 \text{ min})$$

SOL : La durée sera env. 7 minutes 20 s.