Analyse - ch.2 et 3 : limites et asymptotes

Exercice 1. (2+2+2=6 pts)

Série A

a) $\lim\limits_{x\to-2} \dfrac{x+2}{x^2-4} = \lim\limits_{x\to-2} \dfrac{x+2}{(x+2)(x-2)} =$
$= \text{"}\dfrac{0}{0}\text{"} = \lim\limits_{x\to-2} \dfrac{1}{x-2} = -\dfrac{1}{4}$

b) $\lim\limits_{x\to-1} \dfrac{x^2+x-1}{x^2+2x+1} = \lim\limits_{x\to-1} \dfrac{x^2+x-1}{(x+1)^2} =$
$= \text{"}\dfrac{-1}{0_+}\text{"} = -\infty$

c) $\lim\limits_{x\to-\infty} \dfrac{3x^2-2x+1}{x^3+9x^2-73x+99} = \lim\limits_{x\to-\infty} \dfrac{3x^2}{x^3} =$
$= \lim\limits_{x\to-\infty} \dfrac{3}{x} = 0$

Série B

a) $\lim\limits_{x\to-3} \dfrac{x+3}{x^2-9} = \lim\limits_{x\to-3} \dfrac{x+3}{(x+3)(x-3)} =$
$= \text{"}\dfrac{0}{0}\text{"} = \lim\limits_{x\to-3} \dfrac{1}{x-3} = -\dfrac{1}{6}$

b) $\lim\limits_{x\to-1} \dfrac{x^2+x+1}{x^2+2x+1} = \lim\limits_{x\to-1} \dfrac{x^2+x+1}{(x+1)^2} =$
$= \text{"}\dfrac{1}{0_+}\text{"} = +\infty$

c) $\lim\limits_{x\to-\infty} \dfrac{4x^2-3x+2}{x^3+7x^2-65x+98} = \lim\limits_{x\to-\infty} \dfrac{4x^2}{x^3} =$
$= \lim\limits_{x\to-\infty} \dfrac{4}{x} = 0$

Exercice 2. (1+3+1=5 pts)

Série A

a) $f(x) = \dfrac{x(x-1)}{(x-1)(x-2)} \Rightarrow ED_f = \mathbb{R} \setminus \{1;2\}$

b) $\lim\limits_{x\to1} f(x) = \text{"}\dfrac{0}{0}\text{"} = \lim\limits_{x\to1} \dfrac{x}{(x-2)} = -1 \Rightarrow$
$\Rightarrow$ trou en $(1;-1)$

$\lim\limits_{\substack{x\to2\\<}} f(x) = \lim\limits_{\substack{x\to2\\<}} \dfrac{x}{(x-2)} = \text{"}\dfrac{2}{0_-}\text{"} = -\infty$
$\lim\limits_{\substack{x\to2\\>}} f(x) = \lim\limits_{\substack{x\to2\\>}} \dfrac{x}{(x-2)} = \text{"}\dfrac{2}{0_+}\text{"} = +\infty \Rightarrow$
$\Rightarrow$ asymptote verticale : $x = 2$

c) $\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} \dfrac{x^2}{x^2} = 1 \Rightarrow$
$\Rightarrow$ asymptote horizontale : $y = 1$

Série B

a) $f(x) = \dfrac{x(x-2)}{(x-2)(x-3)} \Rightarrow ED_f = \mathbb{R} \setminus \{2;3\}$

b) $\lim\limits_{x\to2} f(x) = \text{"}\dfrac{0}{0}\text{"} = \lim\limits_{x\to2} \dfrac{x}{(x-3)} = -2 \Rightarrow$
$\Rightarrow$ trou en $(2;-2)$

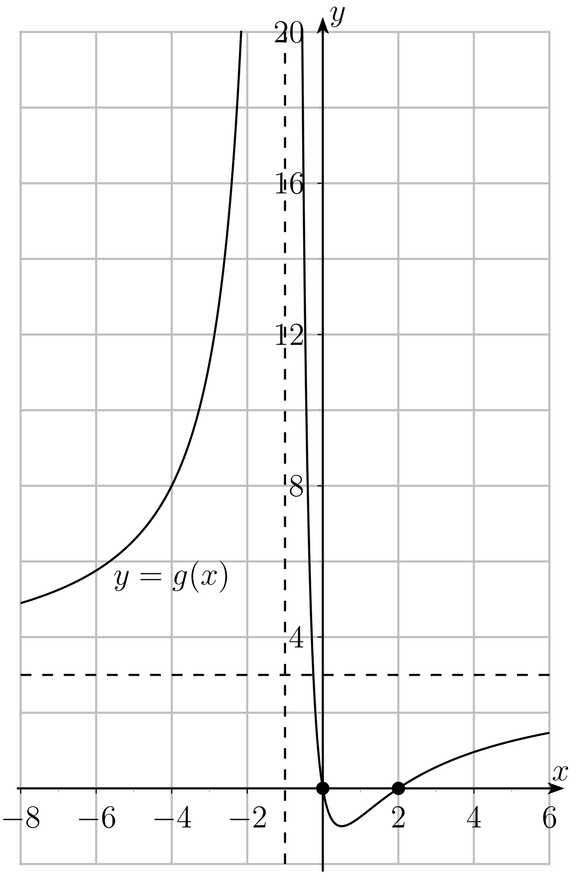
$\lim\limits_{\substack{x\to3\\<}} f(x) = \lim\limits_{\substack{x\to3\\<}} \dfrac{x}{(x-3)} = \text{"}\dfrac{3}{0_-}\text{"} = -\infty$
$\lim\limits_{\substack{x\to3\\>}} f(x) = \lim\limits_{\substack{x\to3\\>}} \dfrac{x}{(x-3)} = \text{"}\dfrac{3}{0_+}\text{"} = +\infty \Rightarrow$
$\Rightarrow$ asymptote verticale : $x = 3$

c) $\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} \dfrac{x^2}{x^2} = 1 \Rightarrow$
$\Rightarrow$ asymptote horizontale : $y = 1$

Exercice 3. (3 pts)

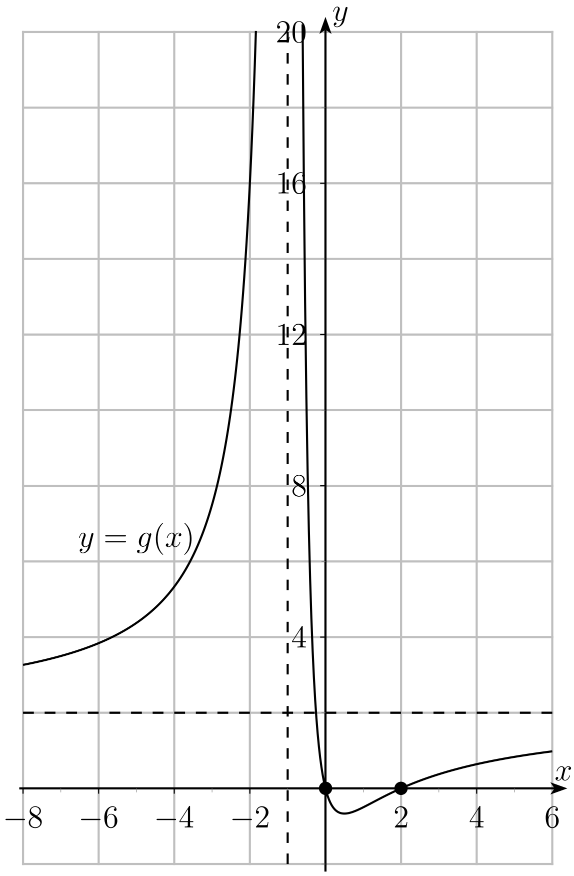
- $Z_g = \{0\,;2\}$
- AV : $x = -1$
- AH : $y = 3$

par exemple : $g(x) = \dfrac{3x(x-2)}{(x+1)^2}$



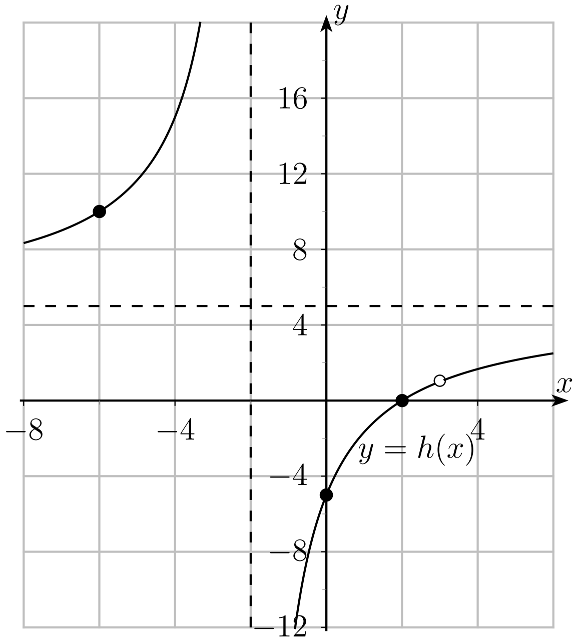
- $Z_g = \{0\,;2\}$
- AV : $x = -1$
- AH : $y = 2$

par exemple : $g(x) = \dfrac{2x(x-2)}{(x+1)^2}$

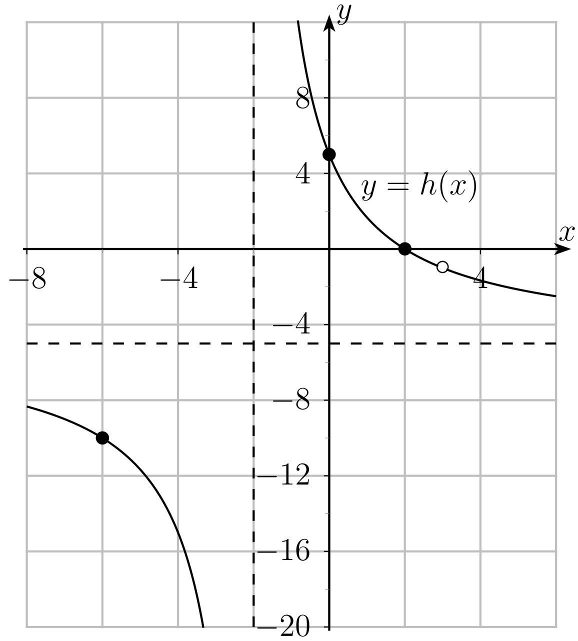


Exercice 4. (3 pts)

par exemple : $h(x) = \frac{5(x-2)(x-3)}{(x+2)(x-3)}$



par exemple : $h(x) = -\frac{5(x-2)(x-3)}{(x+2)(x-3)}$



Exercice 5. (3 pts)

division euclidienne :

$-x^3 + x^2 = (-x+1)(x^2-9) + (-9x+9) \Rightarrow$

$\Rightarrow k(x) = (-x+1) + \frac{-9x+9}{x^2-9}$

$\Rightarrow$ asymptote oblique : $y = -x + 1$

division euclidienne :

$-x^3 + x^2 = (-x+1)(x^2-4) + (-4x+4) \Rightarrow$

$\Rightarrow k(x) = (-x+1) + \frac{-4x+4}{x^2-4}$

$\Rightarrow$ asymptote oblique : $y = -x + 1$